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HOLOMORPHIC FUNCTIONS OF FINITE ORDER ON AFFINE VARIETIES

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Introduction. Holomorphic functions of finite order and their value distribution theory have long been a subject of interest and importance in complex analysis, both in one and several variables. Griffiths [6] and Cornalba–Griffiths [2] outlined a program whose thesis is that analytic objects of finite order provide the "right" setting for doing complex analysis on affine algebraic varieties. The main purpose of this paper is to show that the sheaf of germs of holomorphic functions of finite order $\mathcal{O}_{\text{f.o.}}$ on $\bar{A}$ where A is affine algebraic is flat over the sheaf of germs of holomorphic functions $\mathcal{O}$ on $\bar{A}$. As application we show that for any coherent analytic sheaf $\mathfrak{J}$ over A which extends as coherent sheaf to $\bar{A}$ then $H^q(\bar{A}, \mathfrak{J}_{\text{f.o.}}) = 0$ for all $q \geqslant 1$ where $\mathfrak{J}_{\text{f.o.}} = \mathfrak{J} \otimes \mathcal{O}_{\text{f.o.}}$. Applying this result to the ideal sheaf we show that every holomorphic function of finite order on an affine subvariety of $\mathbf{C}^N$ extends to a holomorphic function of fine order on $\mathbf{C}^N$. This last result was announced by Narasimhan in 1970 [15]. Recently C. Baernstein and A. Taylor gave a proof of this result using techniques of Ehrenpreis.

Our proof is based on the papers of Siu [13] and [14] and of course the standard arguments of flat modules as in Serre ([11] and [12]) and Douady ([3]).

Originally, we assume that A is smooth and the divisor at infinity $D = \bar{A} - A$ has normal crossing (this is the situation considered in Cornalba–Griffiths [2]). The referee pointed out that these restrictions can be removed by modifying a lemma in Gunning and Rossi [8]. We wish to express our gratitude to the referee for the improvement.

§1. Flatness of $\mathcal{O}_{\text{f.o.}}$. Let $V \subset \mathbf{C}^N$ be a subvariety germ at 0, φ a polynomial germ on $\mathbf{C}^N$ at 0 vanishing at 0 but $\not\equiv 0$ on any branch of V. Let H be the zero set of φ. A holomorphic function germ f on $V \setminus H$ at 0 is said to be of *finite order* if there exists positive numbers λ and c such that

$$|f| \leqslant \exp(c|\varphi|^{-\lambda}) \tag{1.1}$$

THEOREM 1.1. *The ring of all holomorphic function germs of finite order $\mathcal{O}_{\text{f.o.}}$ on $V \setminus H$ at 0 is flat over the ring of all holomorphic function germs $\mathcal{O}$ on V at 0.*

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