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IRREDUCIBLE CONSTITUENTS OF PRINCIPAL SERIES OF $SL_n(k)$

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§1. Introduction. The unitary principal series of the general linear group $GL_n(k)$ or the special linear group $SL_n(k)$ over a nondiscrete locally compact field k of characteristic zero consists of the representations unitarily induced from a continuous unitary character of the upper triangular group. In the case of $GL_n(k)$, Gelfand and Naimark [4] gave a proof that shows that these representations are always irreducible (see also [2]).

Our concern in this paper will be with $SL_n(k)$, where reducibility can occur. We shall describe the irreducible constituents of the unitary principal series of $SL_n(k)$, and we shall relate the reducibility that occurs to the abelian Galois extensions of k. In particular, the irreducible constituents will be parametrized by an abelian Galois group.

Some historical remarks about $SL_n(k)$ will put matters in perspective. For $k = \mathbf{C}$, the proof of irreducibility for $GL_n(\mathbf{C})$ given by Gelfand and Naimark [4] also proves irreducibility for $SL_n(\mathbf{C})$. For $SL_n(\mathbf{R})$, reducibility into two pieces can occur [8], and the irreducible constituents were described in [9]. The method of accounting for the reducibility within $SL_n(\mathbf{R})$ turns out to be a prototype for the classification of irreducible tempered representations of real semisimple groups.

For the case that k is nonarchimedean, Winarsky [18] showed that reducibility into more than two pieces can occur. Howe and Silberger [5] proved that in any event the irreducible constituents all have multiplicity one. Muller [14] and Winarsky [18] independently introduced a finite group, known as the R group, to parallel the case of real groups and obtained, with the aid of a completeness theorem due to Harish-Chandra ([17], Theorem 5.5.3.2), a basis for the commuting algebra. Keys [7] clarified the nature of this basis.

In all this, however, the problem of describing the irreducible constituents remained unsolved. Our intention is to give such a description in this paper for k nonarchimedean.

From [3] this result is known already for the case $n = 2$, but our proof is new even in that case. Briefly we start with a character χ_s of the upper triangular group of $SL_n(k)$, extend it to the upper triangular group of $GL_n(k)$ in a particular way, and use the extension to define a group G_χ intermediate between $SL_n(k)$ and $GL_n(k)$. Using an easy general argument, we show that none of the reducibility is lost in passing from $SL_n(k)$ to G_χ. We can then apply a slight

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