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A SUFFICIENT CONDITION FOR THE AVOIDANCE OF SETS BY MEASURE PRESERVING FLOWS IN $\mathbb{R}^n$

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I. Introduction

Dealing with dynamical systems represented by measure preserving flows one often encounters situations in which the flow is defined only locally, on the complement of a "small" set A (see remark 5). It is then important to know whether the set of points whose orbits lead to A in a finite time has zero measure. This, of course, is not implied by the vanishing of the measure of A, which suffices to make A irrelevant for the discussion of various "equilibrium phenomena" in some systems of physical interest.

The main result presented here is a condition which assures the avoidance of sets of sufficiently small "dimension" (defined by the Minkowski contents) by a measure preserving flow whose velocity field has some finite moments. In a previous work it was shown that for any "dimension" above the bound discussed here there are examples of opposite situations (see remark 2).

II. Setup

Definition 1.

1) Let A be a compact subset of $\mathbb{R}^n$. A *local flow* in $\mathbb{R}^n \setminus A$, with the ceiling functions $-\infty \leq S_- < 0 < S_+ \leq \infty$, is a one parameter family of measurable mappings T_t such that

i) $T_t : \{x \in \mathbb{R}^n \setminus A \mid S_-(x) < t < S_+(x)\} \to \mathbb{R}^n \setminus A$

ii) $S_{(\pm)}(T_t x) = S_{(\pm)}(x) - t$

iii) $T_0 x = x$ and $T_{t_1} T_{t_2} x = T_{t_1 + t_2} x$ whenever $t_1,\ t_1 + t_2 \in (S_-(x), S_+(x))$

iv) $\forall\ x \in \mathbb{R}^n \setminus A$, $T_t x$ is continuous in t for $t \in (S_-(x), S_+(x))$

v) If $A \neq \varnothing$ and for some x $|S_{(\pm)}(x)| < \infty$ then

$$\liminf_{t \to S_{\underset{(-)}{+}}(x)} \text{dist.}(T_t x, A) = 0.$$

2) A local flow is *measure preserving (m.p.)* if the mappings T_t preserve the Lebesque measure.

Definition 2. $v : \mathbb{R}^n \to \mathbb{R}^n$ is the *velocity field* of a local flow T_t if for a.e.x

$$T_t x = x + \int_0^t du\ v(T_u x) \qquad \forall\ t \in (S_-(x), S_+(x)). \tag{1}$$

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