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THE NON-DEGENERATE LIMIT FOR SUPERCRITICAL BRANCHING DIFFUSIONS

H. HERING

The aim of this paper is a more complete limit theory for indecomposable, supercritical Markov branching processes with a general set of types. The theory presented in [1, 2] has two significant gaps: The limit degenerates, unless an additional moment condition is satisfied, and no further information on the limit distribution is provided, except its first moment. Here we try to overcome these deficiencies. To avoid technical conditions, we work in the setting of multi-group branching diffusions, already adopted in [16]. However, it is not difficult to extract a more abstract formulation in the style of [15].

The intuitive background is the following. Let X be a domain in a differentiable manifold. Consider an unordered, finite population of particles independently moving on X, each according to the same diffusion process $\{x_t, P^x\}$. Let $\hat{X}$ be the set of finite populations of particles with positions (*types*) in X, including the empty population θ. Suppose that the motion of a particle is stopped between times t and $t + \Delta$ with probability $k(x_t)\Delta + o(\Delta)$, where k is a measurable function on X. If stopped at x, a particle is instantaneously replaced by a finite population of new particles with types in X, according to a probability distribution $\pi(x, \cdot)$, where π is a transition kernel from X to $\hat{X}$. The new particles move and reproduce (*branch*) according to the same rules. The (not necessarily conservative) Markov process $\{\hat{x}_t, \mathbf{P}^{\hat{x}}\}$ in $\hat{X}$, describing the development of the total population in time, is termed *branching diffusion*. We are interested in the behavior of such processes, as $t \to \infty$.

We assume that X is bounded and that the differential generator of the diffusion transition semigroup is uniformly elliptic. We admit mixed boundary conditions, allowing for absorbing, reflecting, and elastic barriers at the boundary. The termination density k is assumed to be bounded, and on the branching law π we impose a first moment and irreducibility condition: Denoting by $\hat{x}[A]$ the number of particles in $\hat{x} \in \hat{X}$ whose type is in $A \subset X$ and defining

$$\hat{x}[\xi] := \int_{\hat{X}} \xi(y)\hat{x}[dy]$$

for every measurable function ξ on X, we assume that

$$\sup_{y \in X} \int_{\hat{X}} \pi(y, d\hat{x})\hat{x}[X] < \infty$$

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