Vol. 44, No. 4 DUKE MATHEMATICAL JOURNAL© December 1977

HOMOTOPY INVARIANCE OF EXT($\mathscr{A}$)

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1. Introduction.

Let $\mathscr{H}$ be a fixed, separable, infinite dimensional, complex Hilbert space, and let $\mathscr{K}(\mathscr{H})$ be the ideal of all compact operators on $\mathscr{H}$.

Our main purpose in this paper is to prove that a functor Ext$(\cdot)$ defined on the category of all quasidiagonal nuclear separable C^*-algebras is homotopy invariant, where Ext($\mathscr{A}$) is the set of all equivalence classes of extensions of $\mathscr{K}(\mathscr{H})$ by the C^*-algebra $\mathscr{A}$.

The first theorem along this line was proved by Brown, Douglas and Fillmore, who showed that the above result is valid on the category of abelian separable C^*-algebras (cf. [8, §2]). Our arguments are based on an idea originated in a construction of [24, §3], where it was shown that the direct sum of a simple unilateral shift and a normal operator whose spectrum is the unit disk is a quasidiagonal operator. Subsequently, in [16] it was proved that the above operator is actually normal plus compact, settling one the starting points of the Brown, Douglas and Fillmore theory. At that time, it was not clear whether the techniques of [24] could be pushed further to prove the result of [16]. Sometime after the Operator theory proof of the fact that Ext($\mathscr{A}$) is a group for abelian C^*-algebras (which was first proved in [6] using methods from Algebraic topology) appeared in [3], it became apparent to us that the techniques of [24] could indeed be pushed further and be used to prove the homotopy invariance result of Brown, Douglas and Fillmore mentioned previously, using the notion of quasidiagonality and the group property of Ext($\mathscr{A}$). In [23] O'Donovan independently discovered that this could be done, and, as an important particular case, presented a proof of the fact that an essentially normal operator whose essential spectrum is the unit disk must be normal plus compact, giving an alternative Operator theory proof of the result of [16]. In the present paper we extend the result of Brown, Douglas and Fillmore mentioned above to the category of quasidiagonal, nuclear, separable C^*-algebras, and some of our arguments contain ideas similar to those used by O'Donovan. In order to state our results more precisely we need to introduce some terminology.

Let $\mathscr{L}(\mathscr{H})$ be the algebra of all bounded operators on $\mathscr{H}$ and let $\pi : \mathscr{L}(\mathscr{H}) \to \mathscr{Q}(\mathscr{H})$ be the canonical quotient map onto the Calkin algebra $\mathscr{Q}(\mathscr{H}) = \mathscr{L}(\mathscr{H})/\mathscr{K}(\mathscr{H})$. Given a separable C^*-algebra $\mathscr{A}$, we denote by $\tilde{\mathscr{A}}$ the C^*-algebra that consists of joining to $\mathscr{A}$ an identity whenever $\mathscr{A}$ is not unital, and let $\tilde{\mathscr{A}} = \mathscr{A}$ if $\mathscr{A}$ is unital. Let **S** be the category of all separable C^*-algebras where the set

Received February 26, 1977. Revision received August 15, 1977. This research was partially supported by a grant of the National Science Foundation.