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QUASIDIAGONALITY IN THE BROWN-DOUGLAS-FILLMORE THEORY

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Introduction

In their recent fundamental papers [2], [3], [4], L. G. Brown, R. G. Douglas and P. A. Fillmore have given a very striking application of the ideas and methods of Algebraic Topology to Operator Theory. The main problem considered there is that of classifying C^*-algebras of operators which commute modulo the compact operators. Their results not only find a highly elegant expression in the language of algebraic topology but have consequences and applications in that field of study. In such an algebraic approach some of the purely operator theoretic concepts are not immediately apparent. In particular, some of the methods do not have immediate extensions to the case of non Abelian C^*-algebras. The motivation then for this paper was to develop proofs which were more operator theoretic in character.

The methods developed here rely mainly on two characteristics of extensions by $C(X)$. One is that these extensions form a group and the other is that $C(X)$ is a "quasidiagonal C^*-algebra". The work of M. D. Choi and E. G. Effros [5], and D. Voiculescu [12], has shown that Ext $\mathscr{A}$ is a group for a wide class of C^*-algebras $\mathscr{A}$. N. Salinas independently obtained the appropriate generalisation of Homotopy Invariance and has used the techniques of this paper to simplify some of his proofs [11]. This completes a certain circle since one of the ingredients in this paper originated in the work of C. Pearcy and N. Salinas [10].

The history of the Brown-Douglas-Fillmore theory and some of the reasons for its importance are outlined in their works. A few of the details will hopefully serve to place the role of quasidiagonality in perspective. In 1909, H. Weyl showed that every self adjoint operator, all operators are assumed bounded, is the sum of a diagonal one and a compact one [13]. In 1935, J. Von Neumann used this result to show that two such operators are unitarily equivalent modulo the compacts if and only if they have the same essential spectrum [9]. In 1970, P. R. Halmos noted that the corresponding results for normal operators were open [8], and used the concept of quasidiagonality which he had earlier introduced [7], to give another proof of the result of Weyl's. Shortly after, I. D. Berg established that every normal operator is of the form diagonal + compact [1], and the characterization of unitary equivalence modulo the compacts was

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