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p-ADIC HYPERGEOMETRIC FUNCTIONS AND THEIR COHOMOLOGY

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O. Introduction

The present work is a generalization to higher dimensions of some recent work of Dwork [11]. In [11], Dwork shows that the same essential techniques used earlier [5] to construct a p-adic cohomology theory for hypersurfaces may also be used to associate cohomology spaces to the p-adic Bessel function. The eigenvalues of the Frobenius action on cohomology are related [5] to exponential sums over finite fields basically by Dwork's trace formula [4], the starting point for his p-adic study of the zeta-function. Let us fix notation. For any Laurent-polynomial, $g(t) \in \mathbb{F}_q[t_1, \cdots t_n, (t_1 \cdots t_n)^{-1}]$, denote by $S_m(g)$ the following exponential sum

$$S_m(g) = \sum \exp\left(\frac{2\pi i}{p} Tr_m g(t)\right)$$

where $Tr_m : \mathbb{F}_{q^m} \to \mathbb{F}_p$ ($p = \text{char } \mathbb{F}_q$) is the absolute trace and the sum runs over all n-tuples, $(t_1, \cdots t_n) \in (\mathbb{F}_{q^m}{}^*)^n$. Denote by $L(g, s)$ the associated L-series

$$L(g, s) = \exp\left(\sum_{m=1}^{\infty} \frac{S_m(g)\, s^m}{m}\right)$$

This L-function is rational and may be written

$$L(g, s) = \frac{\prod_{i=1}^{r} (1 - s\,\eta_i)}{\prod_{i=1}^{r'} (1 - s\,\omega_i)}$$

or equivalently

$$S_m(g) = \omega_1{}^m + \cdots + \omega_{r'}{}^m - \eta_1{}^m - \cdots - \eta_r{}^m .$$

In the case of the Bessel function, the exponential sum in question is the Kloosterman sum, $S_m(f_a{}^{(1)})$, where $\alpha \in \mathbb{F}_q$, $f_a{}^{(1)}$ is the Laurent polynomial, $f_a{}^{(1)}(t) = t + (a/t)$, and $L(f_a{}^{(1)}, s)^{(-1)^{n+1}}$ is a quadratic polynomial whose reciprocal roots $\{\gamma_i\}_{i=1}^2$ are the eigenvalues of Frobenius acting on the Dwork cohomology space. In [21], Weil derives from his result, the Riemann hypothesis for curves, sharp estimates for the complex size of the Kloosterman sum. In

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