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POSITIVE DEFINITE DISTRIBUTIONS ON UNIMODULAR LIE GROUPS

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1. Introduction.

In [5] L. Schwartz proves that any positive definite distribution T on $\mathbf{R}^n$ can be expressed as a finite sum of derivatives of bounded functions. The importance of the result lies in the fact that T is then easily seen to be tempered, and Fourier analysis can be used to prove the Bochner theorem: there is a unique tempered measure μ on $\mathbf{R}^n$ such that $T[\varphi] = \int \hat{\varphi}(x)\, d\mu(x)$ for all test functions φ, where $\hat{\varphi}$ is the Fourier transform of φ.

In this paper we generalize the first stated result to the case where T is a positive definite distribution on a separable unimodular Lie group. The theorem states that any such distribution may be written as a finite sum of (mixed) left and right invariant differential operators appled to bounded functions. The proof, which is pure functional analysis, essentially parallels the euclidean case.

We would like to show that this implies T is "tempered". Unfortunately this is not possible in our general setting as the concept of a "rapidly decreasing function" is very hard to define. This problem has recently been overcome in the semi-simple case, where indeed there exists a whole collection of Schwartz-type spaces. In a forthcoming paper (to appear in the *Journal of Functional Analysis*) we shall show how the results proved here lead to a Bochner theorem for the spherical transform on a semi-simple Lie group.

2. Notation and preliminaries.

(a) General notation. The standard symbols $\mathbf{Z}$, $\mathbf{R}$ and $\mathbf{C}$ shall be used for the integers, the real numbers and the complex numbers, respectively. If S is a set, T a subset and f a function on S, the restriction of f to T is denoted $f|_T$.

If S is a topological space, then the space of continuous functions from S to $\mathbf{C}$ is denoted by $C(S)$, $C_c(S)$ the subset of those of compact support. The support of any $f \in C(S)$ is denoted by supp f. Cl(A) is the closure of a subset A of S.

For E a topological vector space, E' denotes its continuous dual.

(b) Positive definite functions. Let G be an arbitrary group with identity e,

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