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INVARIANT REGULAR IDEALS IN BROWN-PETERSON HOMOLOGY

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1. Introduction. The coefficient ring $BP_* = \pi_*(BP)$ of Brown-Peterson homology is a polynomial ring

$$BP_* = \mathbf{Z}_{(p)}[v_1, v_2, \cdots]$$

where v_i has degree $2(p^i - 1)$ and $\mathbf{Z}_{(p)}$ denotes the integers localized at the prime p [2, 4]. It is often convenient to put $v_0 = p$.

In Brown–Peterson homology and cohomology one has stable operations r_E indexed by exponent sequences $E = (e_1, e_2, \cdots)$. In particular, the r_E act on BP_*, and an ideal I in BP_* is called *invariant* if $r_E(I) \subset I$ for all the operations r_E.

One knows [4, 6] that the only invariant *prime* ideals in BP_* are those of the form $I_0 = 0$, $I_n = (v_0, \cdots, v_{n-1})$ for $n = 1, 2, 3, \cdots$, and $I_\infty = (v_0, v_1, \cdots) = \bigcup I_n$.

On the other hand, once one gets away from prime ideals, one no longer has much control over invariant ideals. Our aim here is to show that one can get a good grasp on the family of invariant *regular* ideals in BP_*. These ideals are only slightly more general than the invariant ideals of the form $(p^{a_0}, v_1^{a_1}, \cdots, v_n^{a_n})$ considered recently by David Baird [3].

Henceforth, all ideals are to be graded and all elements are to be homogeneous.

2. Statement of results. Let $\alpha_0, \cdots, \alpha_n$ be a sequence of elements of BP_*. We call the sequence *invariant* if $r_E\alpha_0 = 0$ for $E \neq 0$ and $r_E(\alpha_i) \in (\alpha_0, \cdots, \alpha_{i-1})$ for $E \neq 0$ and $i > 0$. Notice that in this case, each ideal $(\alpha_0, \cdots, \alpha_i)$ is invariant.

Notice also

(2.1) LEMMA. *If $(\alpha_0, \cdots, \alpha_n)$ is an invariant ideal and*

$$\deg \alpha_0 \leq \deg \alpha_1 \leq \cdots \leq \deg \alpha_n,$$

then $\alpha_0, \cdots, \alpha_n$ is an invariant sequence.

Proof. If $E \neq 0$ then $r_E\alpha_i$ has smaller degree than α_i. The result follows at once.

In particular, each finitely generated invariant ideal in BP_* is generated by an invariant sequence.

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