TENSOR PRODUCTS OF SPACES OF ALMOST PERIODIC FUNCTIONS

H. D. JUNGHENN

1. Introduction. It is shown that if $s \rightarrow u(s)$ is a representation of a semitopological semigroup S over a locally convex space E, then a vector x in E is almost periodic with respect to $u(S)$ if and only if $u(s)x$ can be uniformly approximated by finite sums $\sum f_i(s)x_i$, where the f_i are scalar-valued almost periodic functions on S and $x_i \in E$. This result is used to express the space of E-valued almost periodic functions on S as a topological tensor product of E and the space of all scalar-valued almost periodic functions on S. Similar results are obtained for the weakly almost periodic and strongly almost periodic cases. Finally, the space of almost periodic functions on the Cartesian product of two semigroups is expressed as a topological tensor product of the corresponding spaces of almost periodic functions on the individual semigroups.

2. Preliminaries. Let S be a semitopological semigroup, that is, a semigroup with identity 1 and a topology relative to which multiplication in S is separately continuous. A *representation of S on a locally convex space E* is a homomorphism $s \rightarrow u(s)$ of S into the semigroup $L(E)$ of continuous linear operators on E such that $u(1)$ is the identity operator. A representation u is said to be (*weakly*) *continuous* if $u(s)x$ is a (weakly) continuous function of s for each $x \in E$. A vector $x \in E$ is (*weakly*) *almost periodic* with respect to a representation u if its orbit $O(x) = \{u(s)x : s \in S\}$ is relatively (weakly) compact in E. We shall denote the weak closure of $O(x)$ by $\bar{O}(x)$.

Given a representation u of S on E we shall let E_w and E_a denote respectively the set of weakly almost periodic and almost periodic vectors in E. We also define two other subsets of E:E_0 is the set of all $x \in E_w$ such that $O \in \bar{O}(x)$; E_p is the closed linear span of all the *unitary subspaces* of E, that is the finite dimensional u-invariant subspaces F of E such that the semigroup of operators $u(S)$ restricted to F is contained in a bounded group of operators in $L(F)$ (with identity the identity operator.) The vectors in E_0 are called *flight vectors*, while E_p is the space of *strongly almost periodic vectors*. (see [4, 7]).

Both E_w and E_a are easily seen to be u-invariant linear subspaces of E, hence the restrictions of u to E_w and E_a are representations of S on these spaces. We shall denote these representations by u_w and u_a respectively. If $u_w(S)$ is an equicontinuous family in $L(E_w)$ (as is the case if, for example, E_w is barrelled) and if E is also complete, then E_w and E_a are closed [1] and we have the obvious inclusions $E_p \subset E_a \subset E_w$. Moreover, exactly as in [4], with respect to the

Received September 28, 1973. Revised version received May 24, 1974.