ALMOST ISOMETRIES OF BANACH SPACES AND MODULI OF RIEMANN SURFACES

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1. Introduction and summary. Let $\mathcal{S}$ denote the set of compact bordered Riemann surfaces. For $\bar{S}$ in $\mathcal{S}$ denote the interior of $\bar{S}$ by S and the boundary of $\bar{S}$ by $\partial\bar{S}$. $\bar{S}$ and $\bar{S}'$ in $\mathcal{S}$ will be called conformally equivalent if S and S' are. For $\bar{S}$ in $\mathcal{S}$ the Riemann space of $\bar{S}$, denoted $R(\bar{S})$, is the set of conformal equivalence classes consisting of elements of $\mathcal{S}$ homeomorphic to S. The Teichmüller metric $D(\cdot, \cdot)$ on $R(\bar{S})$ is defined as follows. $D(\bar{S}, \bar{S}') = \inf \{\log K :$ there is a quasiconformal homeomorphism of S onto S' with dilitation $K\}$. (For a discussion of the Teichmüller metric in this context see Earle [6]. For a general discussion of quasiconformal maps and Teichmüller theory see Bers [4] and Ahlfors [1].) We will refer to the topology induced on $R(S)$ by this metric as the moduli topology.

For $\bar{S}$ in $\mathcal{S}$ let $A(S)$ be the supremum normed Banach algebra of functions continuous on $\bar{S}$ and analytic on S. For $\bar{S}$ and $\bar{S}'$ in $\mathcal{S}$ let $L(A(S), A(S'))$ be the set of all continuous invertible linear maps from $A(S)$ to $A(S')$. For T in $L(A(S), A(S'))$ set $c(T) = (||T|| \, ||T^{-1}||)^{-1}$. Note that elements of $L(A(S), A(S'))$ are not required to be algebra mappings. For $\bar{S}$ in $\mathcal{S}$ and $\bar{S}_1$ and $\bar{S}_2$ in $R(\bar{S})$ set $d(\bar{S}_1, \bar{S}_2) = \inf \{- \log c(T) : T \text{ in } L(A(S_1), A(S_2))\}$. (If $L(A(S_1), A(S_2))$ is empty, then $d(\bar{S}_1, \bar{S}_2) = \infty$.)

In a previous paper [11] the author proved the following theorem.

THEOREM 1. *If $\bar{S}$ in $\mathcal{S}$ is planar, then $d(\cdot, \cdot)$ is a metric on $R(\bar{S})$. The topology induced by this metric is equivalent to the moduli topology.*

Although the definitions differ, the moduli topology defined above is the same as the m-topology of [11].

In this paper we will prove the following partial extension of Theorem 1 to all of $\mathcal{S}$.

THEOREM 2. *If $\bar{S}$ is in $\mathcal{S}$, then $d(\cdot, \cdot)$ is a metric on $R(\bar{S})$. The topology induced by this metric is coarser than the moduli topology.*

If $\bar{S}$ is homeomorphic to the unit disk, Theorem 2 is trivial. If $\bar{S}$ is homeomorphic to an annulus, then Theorem 2 follows from Theorem 1. Hence we can assume in what follows that the universal covering surfaces of $\bar{S}$ and of $\hat{S}$, the double of $\bar{S}$, are both conformally equivalent to the upper half-plane.

It is immediate that $d(\cdot, \cdot)$ depends only on the conformal equivalence classes of its arguments, is symmetric, is positive semidefinite, and satisfies

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