ADDENDUM TO "FK-SPACES CONTAINING c_0"

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1. Introduction. In this note we shall improve some of the results of our paper [2], in particular Theorems 16 and 24. The former result is improved by using a refinement of the closed graph theorem [2; Theorem 13] (see also [4]), which enables us to replace the assumption of separability of F by the assumption that F contains no subspace isomorphic to m. In the latter result we can replace m_0 by any dense subspace of m, and this leads to an interesting extension of the Bachelis-Rosenthal theorem on biorthogonal systems in separable Fréchet spaces.

We shall use the same notation as that of [2].

2. A closed graph theorem and applications. Let E and F be locally convex spaces and let $T : E \to F$ be a linear map. We shall say that T is *subcontinuous* if whenever $\sum_{i=1}^{\infty} x_i$ is subseries convergent in E, then $\sum_{i=1}^{\infty} Tx_i$ converges in F and

$$\sum_{i=1}^{\infty} Tx_i = T\left(\sum_{i=1}^{\infty} x_i\right).$$

Thus the Orlicz-Pettis theorem may be interpreted as saying the identity map is subcontinuous from the weak topology on E to the original topology.

THEOREM 1. *Let F be a fully complete space containing no subspace isomorphic to m, and suppose $T : E \to F$ has closed graph. Then T is subcontinuous.*

Proof. If T fails to be subcontinuous, then, since T has closed graph, there exists a series $\sum x_i$ which is subseries convergent in E but such that $\sum Tx_i$ fails to converge in F. Since F is complete, we may further suppose that for some continuous semi-norm p on F we may have $p(Tx_n) \geq 1$ for all n. Hence there is an equicontinuous sequence f_n of linear functionals on F such that $f_n(Tx_n) \geq 1$ for all n. We define a map $R : F \to m$ by $Ry = \{f_n(y)\}_{n=1}^{\infty}$; hence R is continuous.

We also define a map $S : m_0 \to E$ by $S(a) = \sum_{i=1}^{\infty} a_i x_i$; the map S is continuous for the norm topology on m. It follows that TS has closed graph, $TS : m_0 \to F$. Since F is fully complete and m_0 is barrelled [3], we may conclude that TS is continuous [5; 116]. As F is complete we may extend TS to a continuous linear map $V : m \to F$. Now consider $RV : m \to m$; we have $||RV(e^{(n)})|| \geq 1$ for all n. Hence by the Orlicz-Pettis theorem $\sum_{n=1}^{\infty} RVe^{(n)}$ cannot converge weakly subseries, and it follows easily that RV is not weakly compact. We

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