COMPATIBILITY THEOREMS FOR INFINITE DIFFERENTIAL SYSTEMS

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1. Introduction. Theorem 3 below is an extension to infinite differential systems of the main theorem of [2]. It represents a generalization of M. Bôcher's Lemma 1 of [1], of W. M. Whyburn's Lemma 2 of [5], and of Theorem 6.13 of W. T. Reid's comprehensive treatment of infinite differential systems [4]. Theorem 2, of which Theorem 3 is a consequence, is also a generalization of their theorems. It is further noted that Corollary 2 and Lemma 3, results of Theorem 2, compare with Reid's Theorem 6.12 upon which his Theorem 6.13 depends, and similarly with Bôcher's Theorem 1 and Whyburn's Lemma 1.

Let p denote a real number with $p > 1$ arbitrary but fixed, and let l_p denote the set of all sequences $u = (u_i) = u_1, u_2, \cdots$ of real numbers for which $\sum_{i=1}^{\infty} |u_i|^p$ converges. The norm $||\cdot||_p$ for the elements u of l_p is taken to be $||u||_p = (\sum_{i=1}^{\infty} |u_i|^p)^{1/p}$. The elements of l_p together with $||\cdot||_p$ are called vectors in l_p or l_p vectors, where the usual notions concerning vectors apply. The set l_q of vectors with associated norm $||\cdot||_q$ complementary to the vectors in l_p is defined as were those in l_p with p replaced by $q = p/(p - 1)$.

All matrices, unless otherwise indicated, are infinite matrices $A = [A_{ij}]$, $i, j = 1, 2, \cdots$, the entries A_{ij} are real numbers, E being used to denote the identity matrix. As in [3], a matrix A is said to be bounded or is called a bounded matrix if and only if there exists a real number K such that

$$\left| \sum_{i,j=1}^{n} A_{ij} u_j v_i \right| \leq K \left(\sum_{j=1}^{n} |u_j|^p \right)^{1/p} \left(\sum_{i=1}^{n} |v_i|^q \right)^{1/q}$$

holds for $n = 1, 2, \cdots$ and for every l_p vector u and for every l_q vector v. In this case A is said to be bounded by K, and K is called a bound of A. The definition implies the columns and rows of a bounded matrix are respectively vectors in l_p and l_q.

If A is a bounded matrix, then A defines a continuous linear mapping f of l_p into l_p by means of $f(u) = Au = (\sum_{j=1}^{\infty} A_{ij} u_j)$. Indeed, if K is a bound of A, then $||Au||_p \leq K\, ||u||_p$. Moreover, A also defines a continuous linear mapping g of l_q into l_q via $g(v) = vA = (\sum_{i=1}^{\infty} A_{ij} v_i)$ with $||vA||_q \leq K\, ||v||_q$. If, on the other hand, it is known that A is a matrix which defines a continuous linear mapping f of l_p into l_p as described above (implying in turn the existence of a K such that $||Au||_p \leq K\, ||u||_p$ for every l_p vector u), then A is bounded by K. The same conclusion follows if it is known that A defines a continuous linear mapping of l_q into l_q with $||vA||_q \leq K\, ||v||_q$. With this then, the norm of a bounded matrix A, $||A||$, is taken to mean the greatest lower bound of the set of all

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