A σ-INITIALLY LOCALLY FINITE CHARACTERIZATION OF LARGE INDUCTIVE DIMENSION FOR METRIC SPACES

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Morita characterized large inductive dimension, Ind, for metric spaces by means of a σ-locally finite base. (It is well known that Ind is equivalent to covering dimension for metric spaces.) Vaughan characterized Ind with a σ-closure preserving base. Fitzpatrick and Ford used the concept of an initially locally finite collection of open sets to find a sufficient condition for the equivalence of small and large inductive dimension for metric spaces. The purpose of this paper is to characterize Ind for metric spaces using a development that is a σ-initially locally finite base.

Definitions and Notations. Let I denote the set of positive integers, and let Q denote the set of integers greater than or equal to -1.

If h is a point set, then $\bar{h}$ denotes the closure of h and $B(h)$ denotes the boundary of h. If H is a collection of point sets, then H^* denotes the union of the elements of H.

If H is an ordered collection of point sets and $g \in H$, then let $I_g(H)$ be the set of elements of H that precede g.

The ordered collection H of point sets is *initially locally finite* means if $h \in H$ then $I_h(H)$ is locally finite at each point of h.

A *development* for a topological space X is a sequence $G_1, G_2, G_3, \cdots$ of open covers of X such that if p is a point and E is an open set containing p, then there is a positive integer n such that $\{g\colon g \in G_n \text{ and } p \in g\}^* \subset E$.

The topological space X possesses property P_{-1} means $X = \phi$. The topological space X possesses property P_j, where j is a nonnegative integer, means there is a development $G_1, G_2, G_3, \cdots$ for X such that if $n \in I$, then (a) there is an order relation O_n according to which G_n is initially locally finite, and (b) if $g \in G_n$ then $B(g)$ possesses property P_{j-1}.

Any definitions, notations, or theorems used without reference are found in [4].

Lemma 1. *If X is a metric space, and* Ind $X \leq k$, *then there is a development $G_1, G_2, G_3, \cdots$ of X such that if $n \in I$, then* (a) *there is an order relation O_n according to which G_n is initially locally finite, and* (b) *if $g \in G_n$, then* Ind $B(g) \leq k-1$.

Proof. Assume X is a metric space and Ind $X \leq k$. Let $H_1, H_2, H_3, \cdots$ be a sequence of open covers of X such that if $n \in I$ and $g \in H_n$, then diam $(g) < 1/n$. Assume $n \in I$. By [2, Corollary] there are $k+1$ discrete collections of open sets $L_1, \cdots, L_{k+1}$ such that (1) each L_i refines H_n, (2) $L_1 \cup L_2 \cup \cdots \cup L_{k+1}$ covers X, and (3) if $g \in L_i$, then Ind $B(g) \leq k - 1$.

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