ON THE FOURIER SERIES OF CERTAIN CHARACTERISTIC FUNCTIONS ON $SU(2)$

By R. A. Mayer

Let $G = SU(2)$ be the group of 2×2 unitary matrices with determinant 1. For each natural number n let E_n be the unique minimal closed two-sided ideal of dimension n^2 in the convolution algebra $L^2(G)$. Let χ_n denote the unique irreducible character in E_n and let P_n denote the orthogonal projection of $L^2(G)$ onto E_n. Then

$$P_n f = n\chi_n * f \tag{1}$$

for all $f \in L^2(G)$, where $*$ denotes convolution. If $f \in L^2(G)$, write

$$S_n f = \sum_{j=1}^{n} P_j f \tag{2}$$

and call $S_n f$ the n-th partial sum of the Fourier series for f. We will say that the Fourier series for f converges to b at x if $\lim_{n\to\infty} S_n f(x) = b$. Let M be a 3-dimensional compact submanifold-with-boundary of G (so that the interior M^0 of M is open in G). Define the *modified characteristic function* ϕ_M of M by

$$\begin{aligned} \phi_M(x) &= 1 \quad \text{if} \quad x \in M^0 \\ &= \tfrac{1}{2} \quad \text{if} \quad x \in \partial M \\ &= 0 \quad \text{if} \quad x \notin M. \end{aligned}$$

Let θ be the function on G defined by

$$\theta(x) = \operatorname{arc\,cos}\left(\tfrac{1}{2} \operatorname{trace} x\right) = \operatorname{arc\,cos}\left(\tfrac{1}{2}\chi_2(x)\right) \tag{3}$$

so that the eigenvalues of x are $\exp(\pm i\theta(x))$. We consider G to be a metric space with the translation invariant metric ρ defined by

$$\rho(x, y) = \theta(xy^*) \tag{4}$$

where y^* denotes the adjoint of the matrix y. Let $S(x, r)$ denote the sphere with center at x and radius r. Then it is easy to verify that $S(x, r) = S(-x, \pi - r)$. We will call both x and $-x$ the *centers* of $S(x, r)$. We have

$$\chi_n(x) = \sin n\theta(x)/\sin \theta(x) \qquad n \geq 1 \tag{5}$$

(by [9; 151 and 163]). We will prove the following result.

Theorem. *Let M be a 3-dimensional compact analytic submanifold-with-boundary of $SU(2)$ and let ϕ_M be the modified characteristic function of M. Then*

Received February 28, 1969.