RESEARCHES ON PARTITIONS

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In this paper we shall find explicit formulas for the number of partitions of n into parts not exceeding m for certain values of m. We denote by $p_m(n)$ the number of partitions of n into parts not exceeding m, where

$$(1) \qquad F_m(x) = 1/(1-x)(1-x^2)\cdots(1-x^m) = \sum_{n=0}^{\infty} p_m(n)x^n,$$

and $p_m(0) = 1$.

It is interesting to note that Cayley [2] found formulas for $m = 1$ through 6 inclusive, but did not go on to $m = 7$. To express $p_7(n)$ by Cayley's method requires the use of 420 polynomials each of degree 6. In fact, to determine explicit formulas for $p_6(n)(n = 0, 1, 2, \cdots)$ Cayley had to use 60 polynomials each of degree 5.

Now, because of the new method introduced in this paper, for the first time, we are able to find explicit formulas for the $p_m(n)$ with values of m that are greater than 6.

We define

$$(2) \qquad A(m, n) = 1 \quad \textit{if } m \textit{ divides } n$$

and

$$A(m, n) = 0 \quad \textit{if } m \textit{ does not divide } n$$

where

$$m = 1, 2, 3, \cdots, \qquad n = 0, 1, 2, \cdots,$$

and

$$A(m, 0) = 1.$$

(For example $A(15, 25) = A(3, 2) = 0$ and $A(11, 22) = A(3, 3) = 1$.)

Now, by (1) it is seen that the product

$$F_m(x) \sum_{n=0}^{\infty} x^{mn+n} = F_{m+1}(x),$$

and by comparing coefficients we have

$$(3) \qquad p_{m+1}(n) = p_m(n) + p_m(n-m-1) + p_m(n-2m-2) + p_m(n-3m-3) \cdots$$

where $m = 1, 2, 3, \cdots$, and $n = 0, 1, 2, \cdots$.

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