O^N-APPROXIMABLE AND HOLOMORPHIC FUNCTIONS ON COMPLEX SPACES

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In [5] Spallek proved the following two statements:

(1) For every point x of an analytic subvariety X of an open subset of $\mathbf{C}^n$ there exist a neighborhood system $\mathfrak{U}$ of x in X and a natural number $N = N(\mathfrak{U})$ depending on $\mathfrak{U}$ such that, if f is a weakly holomorphic function on U for some $U \in \mathfrak{U}$ and Re f is the restriction to U of an N times differentiable function on some open neighborhood of U in $\mathbf{C}^n$, then f is strongly holomorphic on U ([5, Satz 4.2] and Zusatz bei Korrektur).

(2) For every point x of a reduced complex space X there exists a natural number $N = N(x)$ depending on x such that, if f is a weakly holomorphic function-germ at x and Re f is O^N-approximable in some neighborhood of x, then f is a strongly holomorphic function-germ at x ([5, Satz 3.6] and Zusatz bei Korrektur).

A natural question arises: whether $N(x)$ in (2) can be chosen to be locally independent of x. In this paper we prove that this is the case:

THEOREM 1. *For every compact subset K of a reduced complex space X there exists a natural number $N = N(K)$ depending on K such that, if f is a weakly holomorphic function-germ at $x \in K$ and* Re f *is O^N-approximable in some neighborhood of x, then f is a strongly holomorphic function-germ at x.*

Obviously Theorem 1 implies (1) and, moreover, it implies that in (1) $N = N(\mathfrak{U})$ can be chosen to be independent of $\mathfrak{U}$ if every member of $\mathfrak{U}$ is contained in a fixed compact subset of X.

In what follows complex spaces and complex subspaces are in the sense of Grauert [2, §1], i.e. their structure-sheaves may have non-zero nilpotent elements. Subvarieties are subsets of complex spaces defined locally by the vanishing of a finite number of holomorphic functions on their reductions. For a local ring R, $m(R)$ denotes the maximal ideal of R. ${}_n\mathcal{O}$ denotes the structure-sheaf of $\mathbf{C}^n$.

Theorem 1 will be derived from the following theorem which by itself is of interest:

THEOREM 2. *Suppose $\mathfrak{F}$ is a coherent analytic sheaf on a complex space $(X, \mathcal{H})$. Then for every relatively compact open subset Q of X there is a natural number $N = N(Q)$ depending on Q such that, if $s \in \Gamma(U, \mathfrak{F})$ for some open subset U of Q and $s_x \in m(\mathcal{H}_x)^N \mathfrak{F}_x$ for $x \in U$, then $s = 0$.*

For the proof of Theorem 2 we need the following:

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