REPAIRING EMBEDDINGS AND DECOMPOSITIONS IN S^3

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I. Introduction. Hempel has shown that under certain conditions one may change the imbedding of a torus in S^3 with a monotone map of S^3 onto itself which is a homeomorphism on a neighborhood of the torus. In particular, he showed one may unknot, but not knot, a torus with such a map [4].

In §III of this paper we will explore the possibility of modifying the embedding of a Cantor set with a monotone map of S^3 onto itself which is a homeomorphism on the Cantor set and which does not collapse anything onto the Cantor set.

Bing has shown that there is a monotone map of S^3 onto itself which takes a pair of linked simple closed curves onto a pair of points [3]. He does this by constructing a monotone upper semi-continuous decomposition of S^3 which has the linked simple closed curves as nondegenerate elements and whose decomposition space is S^3. The natural projection map is thus the map which does the job.

Bing has also announced his ability to prove that if $C_1, C_2, \cdots, C_N$ is any finite collection of pairwise disjoint nonseparating continua in S^3 then there is a monotone map of S^3 onto itself such that each C_i is the preimage of a distinct point.

In §IV of this paper we extend this result and prove that if G is any monotone upper semi-continuous decomposition of S^3 which is definable by manifolds-with-connected-boundary, then we may add nondegenerate elements to those in G to get a decomposition whose decomposition space is S^3.

In §V we will combine the ideas of §§III and IV to show how it is sometimes possible to repair the imbedding of collections of continua by a monotone map of S^3 onto itself which does not disturb the continua, only the imbedding.

II. Definitions and notation. The notation is standard. See, for example, Armentrout's article in [1]. A decomposition G of S^3 is upper semi-continuous if and only if given an element $g \in G$ and a neighborhood U of g there is a neighborhood V of g so that if $g' \in G$ and $g' \cap V \neq \phi$, then $g' \subset U$. The decomposition space S^3/G is the space whose points are the elements of G and whose open sets are defined as follows. $U \subset S^3/G$ is open if and only if $\bigcup_{g \in U} g$ is open in S^3. The natural projection $\pi: S^3 \to S^3/G$ is defined as follows. $\pi(x) = g$ if and only if $x \in g$. π is continuous (if G is upper semi-continuous). H denotes the collection of nondegenerate elements of G and H^* the union of the nondegenerate elements. A decomposition is definable by manifolds-with-boundary provided there is a sequence M_i of 3-manifolds-with-boundary in S^3 such that

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