ON STARLIKE AND CONVEX FUNCTIONS OF ORDER α

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Introduction. In this paper an elementary method of variation for analytic functions represented by a Stieltjes integral is applied to convex and starlike functions of order α. Rotation and distortion theorems, coefficient theorems and subordination theorems are proved for these classes of functions. Furthermore, since when $\alpha = 0$, these classes reduce to the well-known classes of starlike and convex functions, we reprove, by a single method, many of the known results for these functions.

More restricted classes of starlike and convex functions of order α were first introduced by M. S. Robertson [9]. The starlike functions of order α have recently been studied by A. Schild [10] and T. H. MacGregor [6]. Methods used in this paper have previously been used by this author to study close-to-convex functions [8].

1. Preliminaries. Let D denote the open unit disc, $D = \{z: |z| < 1\}$.

DEFINITION 1. *A function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic in D is said to be starlike of order α where α is fixed and $0 \leq \alpha < 1$ if*

$$\operatorname{Re} \frac{zf'(z)}{f(z)} > \alpha \tag{1}$$

for all z in D.

DEFINITION 2. *A function $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic in D is said to be convex of order α where α is fixed and $0 \leq \alpha < 1$ if*

$$\operatorname{Re} \left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \tag{2}$$

for all z in D.

We denote, for fixed α, the class of all starlike functions of order α by S_α and the class of all convex functions of order α by C_α . S_0 and C_0 are the classical classes of starlike and convex univalent functions in D. It will be necessary to refer to the class $\mathcal{P}$ of functions $P(z) = 1 + \sum_{n=1}^{\infty} b_n z^n$ which are analytic and satisfy $\operatorname{Re} P(z) > 0$ in D.

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