BISIMPLE INVERSE SEMIGROUPS MOD GROUPS

By R. J. Warne

In one of the fundamental theorems of the algebraic theory of semigroups, Rees [2], [4] determined the structure of completely simple semigroups mod groups. We determine the structure of several other classes of bisimple semigroups mod groups. As with completely simple semigroups, we are able to develop a homomorphism theory, a theory of congruences, and a theory of extensions for these classes of semigroups.

In §1, we completely describe mod groups the structure of right cancellative semigroups P with identity whose ideal structure (the partially ordered set of principal left ideals of P ordered with respect to inclusion) is order isomorphic to $(I^0)^n$, where I^0 is the non-negative integers and n is a natural number, under the lexicographic order. We call such semigroups ω^n-right cancellative semigroups. In the case $n = 1$, we obtain a theorem of Rees [5]. In [16], [17], we develop a homomorphism theory for ω^n-right cancellative semigroups.

In §2, we describe completely mod groups the structure of certain classes of bisimple semigroups with identity. Let S be a bisimple semigroup and let E_S denote its set of idempotents. We may partially order E_S in the following manner: if $e, f \in E_S$, $e \leq f$ if and only if $ef = fe = e$. We then say that E_S is under or assumes its natural order. If E_S, under its natural order, is order isomorphic to $(I^0)^n$ under the lexicographic order, we call S an ω^n-bisimple semigroup. Let C_1 be the bicyclic semigroup, i.e. $C_1 = I^0 \times I^0$ under the multiplication $(i, j)(k, s) = (i + k - \min(j, k), j + s - \min(j, k))$. Let X be an arbitrary semigroup. We define $C_1 \circ X$ to be $C_1 \times X$ under the multiplication $((m, n), s)((p, q), t) = ((m, n)(p, q), f(n, p))$ where $f(n, p) = s, t$, or st according to whether $n > p$, $p > n$, or $n = p$ and where juxtaposition denotes multiplication in C_1 and X. We define $C_2 = C_1 \circ C_1$, $C_3 = C_1 \circ C_2, \cdots, C_n = C_1 \circ C_{n-1}$. We characterized C_n in [10] and we called C_n the $2n$-cyclic semigroup in [10]. We show that S is an ω^n-bisimple semigroup if and only if $S \cong G \times C_n$, where G is a group under a suitable multiplication. We first give a proof of this structure theorem by combining the structure theorem for ω^n-right cancellative semigroups with a theorem of Clifford [1]. We then give a proof of the direct part of the theorem utilizing [10, Theorem 2.3]. For $n = 1$, we obtain the structure theorem for bisimple ω-semigroups [6], [10]. For $n = 2$, we obtain a structure theorem which extends Theorem 3.2 of [10]. We determined the congruence relations on an ω^n-bisimple semigroup in [14] and the homomorphism and extension theory for ω^n-bisimple semigroups is developed in [16], [18] and [20]. An ω^n-bisimple semigroup is a bisimple inverse semigroup with identity.

In §3, we describe completely mod groups the structure of certain classes

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