ARITHEMETIC FUNCTIONS IN AN UNUSUAL SETTING II

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Let F denote an arbitrary but fixed field. An arithmetic function is a mapping from the integers into F; usually the arguments are limited to positive or non-negative integers. If f and g are two functions, the sum $h = f + g$ is defined by

$$h(n) = f(n) + g(n). \tag{1}$$

The writer [1] has defined the Lucas product $h = f*g$ by means of

$$h(n) = \sum_{a=0}^{n}{}^{*} f(a)g(n - a), \tag{2}$$

where the summation is restricted to such a that

$$p \nmid \binom{n}{a} ; \tag{3}$$

here p denotes a fixed prime.

We recall that (3) holds if and only if

$$n = n_0 + n_1 p + n_2 p^2 + \cdots \qquad (0 \le n_j < p),$$
$$a = a_0 + a_1 p + a_2 p^2 + \cdots \qquad (0 \le a_j < p),$$

and

$$0 \le a_j \le n_j \qquad (j = 0, 1, 2, \cdots). \tag{4}$$

The algebraic system based on (1) and (2) will be denoted by L_p. The object of this note is to point out the relationship of L_p to certain systems based on other definitions of product.

Consider functions that are now defined only on the positive integers. The sum will again be defined by (1). The *unitary* product is defined by means of

$$h(n) = \sum_{\substack{ab=n \\ (a,b)=1}} f(a)g(b), \tag{5}$$

where (a, b) denotes the greatest common divisor of a, b.

The unitary product was introduced by Vaidyanathaswamy [4] who used the term *compounding*; the present terminology was introduced by Eckford Cohen [2], [3]. The algebraic system based on (1) and (5) may be denoted by U_2.

We shall be interested in the subalgebra V_2 consisting of functions f such that

$$f(n) = 0 \qquad (n \text{ not squarefree}); \tag{6}$$

We show that V_2 and L_2 are isomorphic.

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