SOME SEQUENCES OF RATIONAL NUMBERS RELATED TO THE EXPONENTIAL FUNCTION

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1. Introduction. For $k > 0$, define $A_{k,0}, A_{k,1}, A_{k,2}, \ldots$ by

$$\frac{x^k}{k!}\left(e^x - \sum_{s=0}^{k-1} \frac{x^s}{s!}\right)^{-1} = \sum_{r=0}^{\infty} A_{k,r} \frac{x^r}{r!}. \tag{1.1}$$

The numbers $B_n = A_{1,n}$, known as the Bernoulli numbers, are well known [4]. In particular,

$$A_{1,2n+1} = 0 \qquad (n \geq 1),$$
$$(-1)^{n-1} A_{1,2n} > 0,$$

and by the Staudt-Clausen Theorem,

$$A_{1,2n} = G_{2n} - \sum_{p-1 \mid 2n} \frac{1}{p}$$

where $n \geq 1$, G_{2n} is an integer, and the summation is over all primes p such that $p - 1$ divides $2n$.

The case $k = 2$ has also been studied [3]. It is known that

$$2A_{2,n} \equiv 1 \pmod 4 \qquad (n > 1),$$

and for p an odd prime

$$\frac{p^n}{[n(p-2)]!} A_{2,n(p-2)} \equiv 2^n \pmod p \qquad (n \geq 0),$$
$$\frac{p^{[n/(p-2)]+1}}{n!} A_{2,n} \equiv 0 \pmod p \qquad (n \geq 0).$$

The purpose of this paper is to derive properties of $A_{k,n}$, for $k \geq 1$. From (1.1) we obtain the recurrence formula

$$A_{k,0} = 1, \tag{1.2}$$
$$\frac{A_{k,n}}{n!} = -k! \sum_{r=0}^{n-1} \frac{A_{k,r}}{r!\,(n+k-r)!} \qquad (n > 0),$$

or equivalently

$$\sum_{r=0}^{n} \binom{n+k}{r} A_{k,r} = 0 \qquad (n > 0). \tag{1.3}$$

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