OPEN CONTINUOUS MAPPINGS OF SPACES HAVING BASES OF COUNTABLE ORDER

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1. Introduction. This article is a contribution to the study of open continuous mappings of first countable spaces. The main results explicate the role of completeness and bases of countable order in the study of certain such mappings. They may be summarized as follows.

I. The exhibition of a general invariant, *λ-base,* under open continuous mappings onto T_1-spaces (Theorem 1).
II. The isolation of a simple condition, *uniform monotone completeness*, on open continuous mappings that concerns preservation of bases of countable order (Theorem 2).
III. A characterization of T_1-spaces having bases of countable order as uniformly monotonically complete open continuous images of metrizable spaces (Theorem 6).
IV. A characterization of the regular T_0 open continuous images of metrically topologically complete spaces (Theorem 8).
V. Characterizations of metrically topologically complete spaces and metrizable spaces as certain open continuous images (Corollaries 1.1 and 2.1).
VI. A collection of examples delineating sharply the scope of the theory (§8).

Some of the results given here are extensions, complements, or refinements to results of F. Hausdorff [16], A. H. Stone [25], A. V. Arhangel'skiĭ [6], V. I. Ponomarev [22], and E. A. Michael [18], among others. More detailed comments concerning the relationship to Hausdorff's result are given at the end of §4. §2 relates the main concepts to classical ones.

The very wide applicability of the results depends, in part, on the fundamental role of bases of countable order in the characterizations of metrizable [8] and developable [28] spaces.

2. Definitions and remarks. Some of the major concepts used are defined here, with accompanying remarks relating them to classical concepts.

Definitions.

A space is said to be *essentially* T_1 [28] if and only if every two points of the space have either identical or non-intersecting closures.

A collection of sets is called *perfectly decreasing* [28] if and only if it contains a proper subset of each of its elements.

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