GROUPS OF LINEAR FRACTIONAL TRANSFORMATIONS

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1. Introduction. Let G denote a group of linear transformations T, $Tz = (az + b) : (cz + d)$, $ad - bc = 1$.

A point x in the extended plane is called a *limit point* with respect to a subset A of G, if and only if $T_n a \to x$ for some sequence $\{T_n\}$ of distinct elements of A and some point a. Other points are called *ordinary.*

The set of all limit points is denoted by L and its complement by O.

G is said to be *discontinuous* if and only if O is non-void.

G is said to be *discrete* if and only if no sequence in G converges to the identity I.

The main theorem in this note is the following:

Theorem 1. *If G is discrete and $x \in L$, then G has a sequence $\{T_n\}$ of distinct elements, such that $T_n z \to x$ for every z in the extended plane with the possible exception of x and one other limit point y. The convergence is uniform on every compact set which does not contain x or y.*

This theorem is an extension of the following theorem which is stated and proved in [3; 103]:

Theorem (a). *If $\lambda \in L$, the set Gz is dense at λ: (1) for each ordinary point z, (2) for each z in the extended plane with the possible exception of $z = \lambda$ and of one other point.*

Properties of the set L have been investigated extensively. It is known that L is closed. This theorem (Theorem 2) is proved here by using Theorem 1.

The relation between the set L and the set of points which are fixed under elements of G has been described in [3; 104]. Here are the results:

Theorem (b). *If L is not a single point, it is the closure of the set of fixed points of the hyperbolic or loxodromic elements of G.*

Theorem (c). *If L has more than one limit point and G has parabolic elements, L is the closure of the set of the parabolic fixed points.*

It is added there as an exercise that the requirement in (b) that L is not a single point is not essential and may be omitted.

This description is completed here by proving (Theorem 3) that if G is discrete

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