A NOTE ON UPPER SEMICONTINUOUS DECOMPOSITIONS OF THE n-SPHERE

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Given a collection H of disjoint compact sets in a topological space, we denote by H^* the sum of sets in H. An element g of H is called a *central element* if it contains a limit point of the set $H^* - g$. Otherwise g is called *noncentral.* The collection H is said to be *discrete* if every element of H is noncentral.

Let H_0 be a collection of disjoint compact sets in a space. The collection of all central elements of H_0 is denoted by H_1. For each countable ordinal $\alpha \geq 1$, define H_α as follows: Suppose that the collections H_β are already defined for all ordinals $\beta < \alpha$. Then H_α will consist of all those elements of H_0 which are central in each H_β, $\beta < \alpha$. It is to be noted that $H_\alpha \subset H_\beta$ if $\alpha \geq \beta$. An element g of H_0 is said to be *essential* or *inessential* according as g belongs to H_α for every countable ordinal α or not. Finally, we call H_0 a *countably discrete collection* if there exists a countable ordinal α such that H_α is a null collection.

Let G be an upper semicontinuous decomposition of the n-sphere S^n. The collection of nondegenerate elements of G is denoted by H_0. We show in this paper that every inessential element of H_0 has an open n-cell as complement whenever the decomposition space of G is homeomorphic to S^n. If, in particular, H_0 is countably discrete, then the decomposition space of G is topologically equivalent to S^n when and only when each element of G has an open n-cell complement.

The starting point of our work was to prove a converse of a result due to Bing [1, Theorem 1] and thereby to generalize Theorems 3 and 6 of [4]. Namely, under the condition that G has at most a countable number of nondegenerate elements whose sum is a G_δ-set, we will prove that the decomposition space of G is homeomorphic to S^n if and only if each element of G has an open n-cell as complement. Indeed, this is done by showing that the property of the collection H_0 of nondegenerate elements of G being countably discrete is characterized by the above condition imposed on G.

We begin with some lemmas.

Lemma 1. *Let H_0 be a collection of disjoint compact sets in a space. For each inessential element g of H_0, there exists a unique countable ordinal α such that g is noncentral in H_α. In other words, $H_0 - K$ is the union of disjoint discrete collections $H_\alpha - H_{\alpha+1}$, where K denotes the collection of all essential elements of H_0.*

Proof. Let g be any inessential element. By definition, there exists a countable ordinal β such that g is not an element of H_β. Let β be the least

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