LEFT AND RIGHT BOUNDARY CLUSTER SETS IN n-SPACE

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1. Introduction. Let f be a function from the open unit disk (in the complex plane) into the extended complex plane, and let C denote the unit circle. In 1960 Collingwood [2] proved the following.

THEOREM. *The set of points p on C such that $C_{BL}(f, p)$ is unequal to $C(f, p)$ is at most countable.*

COROLLARY 1. *The set of points p on C such that $C_{BL}(f, p)$ is unequal to $C_{BR}(f, p)$ is at most countable.*

COROLLARY 2. *The set of points p on C such that $C_B(f, p)$ is unequal to $C(f, p)$ is at most countable.*

We shall establish some results similar to these for functions in n-space ($n > 2$).

2. Definitions. Let E^n denote Euclidean n-space, and let $d_n(p, q)$ denote the usual distance between the points p and q in E^n. The open unit ball D^n is the set of all points p in E^n such that $d_n(p, 0) < 1$, and the boundary of D^n will be denoted by B^n. An n-cell is any set homeomorphic to the closure $B^n \cup D^n$ of D^n, and the n-sphere S^n is the one-point compactification of E^n.

Let f be a function from D^n into S^n, and let p be a point in B^n. The *cluster set* $C(f, p)$ of f at p is the set of all points s in S^n such that there exists a sequence $\{z_i\}$ of points from D^n with $z_i \to p$ and $f(z_i) \to s$. The *boundary cluster set* $C_B(f, p)$ of f at p is the set $\bigcap (\bigcup C(f, q))^*$, where the intersection is taken over all neighborhoods N of p and the union over all q in $N \cap B^n$ with $q \neq p$ (the asterisk denotes closure).

Collingwood's proof of the theorem stated in the introduction is easily modified so that it yields the following proposition.

COROLLARY 2′. *The set of points p in B^n such that $C_B(f, p)$ is unequal to $C(f, p)$ is at most countable.*

In order to extend to functions in n-space the definitions of left and right boundary cluster sets, we introduce a notion of leftness and rightness at points in B^n.

Let p be a point in B^n. An $(n - 2)$-cell Γ in B^n will be called a *separating cell* for p if p lies in the combinatorial interior of Γ. Let Γ be a separating cell for p, and let N be a spherical neighborhood of p (in the relative topology of B^n)

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