# MATRIX REPRESENTATIONS OF COMPACT SIMPLE SEMIGROUPS

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Matrix representations of semigroups have been treated in considerable detail, notably by Suschkewitsh [9], Rees [8], Clifford [3], Preston [7], and Munn [5], [6]. A complete treatment of the theory of representations of abstract semigroup is given in [4]. The lack of topological results in this direction stems primarily from the loss of the theory of invariant measures; there is no analogue to the theorem of Peter and Weyl in compact semigroups.

This paper is concerned with faithful topological representations of certain finite dimensional compact simple semigroups as semigroups of matrices over the real or complex numbers. It is shown that any compact simple semigroup $S$ in which the idempotents form a semigroup, and in which the maximal groups are finite, is isomorphically imbeddable in the non-negative real matrices whose order is a function of the dimension of $S$ and the order of a maximal subgroup of $S$. A similar theorem is obtained by replacing "finite" by "Lie" and "non-negative real" by "complex" in the previous sentence.

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The set of order $n$ non-negative real matrices is denoted by $N_n$. The semigroup terminology is that of [10]; in particular, $E$ denotes the set of idempotents of $S$, and for $e \in E$, $H(e)$ is the maximal subgroup of $S$ containing $e$. An iseomorphism is an isomorphism which is also a homeomorphism. The topology of $N_n$ is any locally convex topology; for example, the topology of Euclidean $n^2$-space. The characterization of compact simple semigroups treated by Wallace in [11] is assumed without further reference.

Theorem 1. *Let $S$ be a compact, simple, idempotent semigroup contained in euclidean n-space. Then $S$ is iseomorphically imbeddable in $N_{2n+2}$.*

*Proof.* By the compactness, $S$ is bounded. Hence there exists a homeomorphism of $E^n$ carrying $S$ into the non-negative cone of $E^n$. Assume this has been done. For each $y = (y_i) \in S$, define $A(y)$ to be the $(n + 1) \times (n + 1)$ non-negative real matrix having $1, y_1, \cdots, y_n$ as its first row and zeros elsewhere. Further, let $B(y)$ be the transpose of $A(y)$. Note, for every $y \in S$, $A(y)$ and $B(y)$ are idempotents. Fix $x \in S$. For $y \in xS$, let

$$f(y) = \begin{pmatrix} A(y) & 0 \\ 0 & B(x) \end{pmatrix}.$$

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