RECURRENCE TIMES AND CAPACITIES FOR FINITE ERGODIC CHAINS

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I. Introduction. Let us first recall a few well-known results concerning the connection between potential theory and certain Markov processes.

1. Let $\mathbf{r}(\mathfrak{Z})$ be a Brownian motion in $E^3(\mathfrak{Z} \in [0, \infty), \mathbf{r}(0) = 0)$ and let Ω be a closed, bounded region, whose boundary Σ is smooth. If we denote by $H(\mathbf{y})$ the probability that the Brownian motion $\mathbf{y} + \mathbf{r}(\mathfrak{Z})$ never reaches Ω, it has been shown (see for instance Ito and Mc Kean [3]) that $H(\mathbf{y})$ has the following properties:

$$(1.1) \qquad \begin{aligned} H(\mathbf{y}) &= 1 - \int_\Omega \frac{\mu(d\varrho)}{|\mathbf{y} - \varrho|} \qquad \mathbf{y} \notin \Omega \\ 0 &= 1 - \int_\Omega \frac{\mu(d\varrho)}{|\mathbf{y} - \varrho|} \qquad \mathbf{y} \in \Omega \end{aligned}$$

where $\mu(\cdot)$ is a completely additive non-negative set function with support on Σ. $H(\mathbf{y})$ is harmonic outside Ω, vanishes on Σ and $H(\mathbf{y}) \to 1$ as $|\mathbf{y}| \to \infty$.

Define

$$C(\Omega) = \int_\Omega \mu(d\varrho).$$

It is also known that C satisfies the Kelvin principle, i.e.

$$(1.2) \qquad \frac{1}{C(\Omega)} = \inf \int_\Omega \int_\Omega \frac{\varphi(\mathbf{r})\varphi(\varrho)}{|\varrho - \mathbf{r}|} \, d\varrho \, d\mathbf{r}$$

where inf is taken over all functions $\varphi \in L^2(\Omega)$ such that

$$\int_\Omega \varphi(\varrho) \, d\varrho = 1.$$

$C(\Omega)$ is, in fact, the capacity of Ω and $1 - H(\mathbf{y})$ is the generalized capacitary potential.

2. Let $\mathbf{r}(\mathfrak{Z})$ be a Brownian motion in $E^2(\mathfrak{Z} \in [0, \infty))$. Define Ω as in §I.1 and let the random variable $T(\mathbf{y})$ be the time spent to reach Ω starting from $\mathbf{y}$ at $\mathfrak{Z} = 0$. ($H(\mathbf{y}) = \Pr [T(\mathbf{y}) = \infty]$). We get (Hunt [1])

$$(1.3) \qquad \Pr [T(\mathbf{y}) \geq t] \sim \frac{H(\mathbf{y})}{\log \sqrt{t}} \qquad t \uparrow \infty \qquad \mathbf{y} \notin \Omega$$

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