WEIGHTED TWO-LINE ARRAYS

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1. **Introduction.** The present paper is concerned with the problem of evaluating the sum

$$p(n, m; a) = \sum a^{\sum n_i + \sum m_i}, \tag{1.1}$$

where the outer summation is extended over all two-line arrays

$$\begin{cases} n_1 & n_2 & n_3 & \cdots \\ m_1 & m_2 & m_3 & \cdots \end{cases} \tag{1.2}$$

subject to the conditions $n_1 = n$, $m_1 = m$,

$$n_i > m_i, \quad n_i > n_{i+1}, \quad m_i > m_{i+1}, \quad n_i \geq 0, \quad m_i \geq 0. \tag{1.3}$$

This question was suggested by the following problem.

Let $GF(q)$ denote the finite field of order q and let $GF[q, x]$ denote the domain of polynomials in x with coefficients in $GF(q)$. If $A, B \in GF[q, x]$ we seek the number of partitions

$$A = \sum U_i, \qquad B = \sum V_i, \tag{1.4}$$

where U_i, V_i are normalized polynomials in $GF[q, x]$ such that

$$\deg U_i = n_i, \qquad \deg V_i = m_i, \qquad \deg A = n, \qquad \deg B = m$$

and the n_i, m_i satisfy (1.3). If $P(A, B)$ denotes this number, it is easily seen that

$$P(A, B) = q^{-n-m} p(n, m; q). \tag{1.5}$$

We remark that the corresponding problem for simple partitions [1]

$$A = \sum U_i, \tag{1.6}$$

where

$$\deg U_i = n_i, \qquad \deg A = n = n_1, \qquad n_i > n_{i+1}$$

is not difficult; the number of partitions in question is given by

$$\prod_{r=0}^{n-1} (1 + q^r). \tag{1.7}$$

Returning to $p(n, m; a)$ we remark that the case $a = 1$ has been treated in [2]. Put

$$p(n, m) = p(n, m; 1). \tag{1.8}$$

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