UPPER SEMI-CONTINUOUS DECOMPOSITIONS AND FACTORIZATION OF CERTAIN NON-CONTINUOUS TRANSFORMATIONS

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1. Introduction. This paper is concerned with the relationship between certain non-continuous functions and the decomposition of the domain space into upper semi-continuous collections. Related to this is the factorization of functions and the properties possessed by the factors. For continuous transformations Whyburn, in [5], Chapters VII and VIII, has a thorough exposition. The following theorems will show that many of the classical results for continuous functions have their counterpart for peripherally continuous functions and connectivity maps.

2. Definitions. A mapping $f : S \to T$ is peripherally continuous if for every point p in S and for every pair of open sets U and V containing p and $f(p)$, respectively, there is an open set $N \subset U$ and containing p such that $f(F(N)) \subset V$, where $F(N)$ denotes the boundary of N [1; 751]. The mapping f is a connectivity map if for every connected set A in S the set $g(A)$ is connected, where $g : S \to S \times T$ is the graph map induced by f and defined by $g(p) = (p, f(p))$ [1; 750]. Note that if f is peripherally continuous, then the graph map g is also peripherally continuous and conversely.

A space S is locally peripherally connected if for every point p in S and every open set U containing p there is an open set $V \subset U$ and containing p such that $F(V)$ is connected [4; 252].

A useful characterization of an upper semi-continuous collection in a compact metric space S is as follows: A necessary and sufficient condition that a collection G of closed sets be upper semi-continuous is that for any $\{g_n\}$ of elements of G with $g \cap (\liminf g_n) \neq \phi$, where g is in G, then $\limsup g_n \subset g$ [5; 122].

Throughout this paper, unless otherwise stated, S will denote a locally peripherally connected, compact, separable metric space and T a regular Hausdorff space. The proofs of the following theorems rely heavily upon the fact, proved by J. Stallings [4; 255], that if f is a peripherally continuous mapping of the locally peripherally connected space S into the space T, then for every point p in S and every pair of open sets U and V containing p and $f(p)$, respectively, there is an open connected set $N \subset U$ and containing p such that $F(N)$ is connected and $f(F(N)) \subset V$.

3. Decompositions and factorizations. If f is a continuous function from a space S into a space T and C is a closed subset of T, the set $f^{-1}(C)$ is closed.

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