A CLASS OF DISCRIMINATORY SOLUTIONS TO SIMPLE N-PERSON GAMES

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I. Preliminary. In analyzing the constant-sum 3-person game, it is seen that, apart from the finite, symmetric solution, a family of discriminatory solutions exists. A typical member of this family, denoted by $V_{i,c}$, is the set of all imputations x for which $x_i = c$. The only constraint on this is that c must be smaller than $\frac{1}{2}$ (in $0 - 1$ normalization); otherwise the set $V_{i,c}$ will not be externally stable (i.e., it will not dominate everything outside $V_{i,c}$) and hence will not be a solution.

The question naturally arises as to the existence of such solutions for other simple games (not necessarily superadditive). This can be answered in the affirmative.

THEOREM 1. *Let v be a simple game in* (0, 1)*-normalization, and let S be a minimal winning coalition. Let V_S be the set of all imputations x for which $x_j = 0$, if $j \notin S$. Then V_S is a solution to v.*

Proof. Internal stability: Suppose x and y are both in V_S, and suppose $x \succ y$. This domination must be through some winning coalition T. But T cannot include any members of $N - S$, as, for these, $x_j = y_j$. Neither can $T = S$ as

$$\sum_{j \varepsilon S} y_j = 1 = \sum_{j \varepsilon S} x_j ,$$

and we cannot have $x_j > y_j$ for all $j \varepsilon S$. Finally T cannot be a proper subset of S, as S is a minimal winning set. But there are no other possibilities for T. Hence we cannot have $x \succ y$.

External stability: Let y be an imputation not in V_S. As it is not in V_S, $y_j > 0$ for some $j \notin S$, and hence

$$\sum_{j \varepsilon N-S} y_j = \epsilon > 0.$$

Letting s be the number of elements in S, consider the imputation x given by:

$$x_j = y_j + \epsilon/s \quad \text{for} \quad j \varepsilon S$$

$$x_j = 0 \qquad\qquad \text{for} \quad j \notin S.$$

Clearly $x \varepsilon V_S$. Also, $x \succ y$ through the coalition S. Hence V_S is a solution.

Remark 1. In proving internal stability, use was made only of the fact that,

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