## ALGEBRA OF FORMAL POWER SERIES

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1. **Introduction.** This note contains a generalization to several variables of of the explicit coefficient formulas of the algebra of power series. A sequel will apply them to the solution of non-linear simultaneous equations.

2. **The multinomial theorem.** Lower case Latin letters denote complex numbers or complex-valued functions. A neighborhood in the $n$-dimensional complex vector space $C^n$ is a polycylinder. $\zeta = (z_1, \cdots, z_n)$, $\tau = (t_1, \cdots, t_n)$ and $\omega = (w_1, \cdots, w_n)$ are arbitrary elements of $C^n$. Except for the foregoing modifications this paper will adhere to all the definitions and conventions stated in [1, §2]. In particular, all other lower case Greek letters represent elements of the subset $F_n$ of $C^n$.

If $\mu = (m_1, \cdots, m_n)$ let

$$\zeta^\mu = \prod z_j^{m_j}, \quad \partial_\mu = \frac{\partial^{m_1 + \cdots + m_n}}{\partial t_1^{m_1} \cdots \partial t_n^{m_n}} \quad \text{and} \quad \mu! = \prod m_j!,$$

the product being over all $j$, $1 \leq j \leq n$, such that $m_j \neq 0$. A function $f: C^n \to C^1$ is said to be analytic at $\omega \in C^n$ if it has a Taylor series expansion $f(\zeta) = \sum_\mu a_\mu(\zeta - \omega)^\mu$ which converges in a neighborhood of $\omega$. Note that

$$a_\mu = (1/\mu!)(\partial_\mu f(\tau))_{\tau=\omega} .$$

Let

$$\prod{}' = \prod_{\theta < \lambda \leq \mu} \quad \text{and} \quad \sum{}' = \sum_{\theta < \lambda \leq \mu} .$$

If $v \in B(\mu)$ [1, §2], then by definition

$$a^v = \prod{}' a_\lambda^{v(\lambda)}, \quad |v| = \sum{}' v(\lambda) \quad \text{and} \quad \binom{k}{v}$$

$$= k(k-1) \cdot \cdots \cdot (k - |v| + 1)/\prod{}' v(\lambda)!.$$

Let

$$\binom{\mu}{v} = \mu!/\prod{}' v(\lambda)!.$$

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