ABSOLUTE CONVERGENCE AND THE GENERAL HIGH INDICES THEOREM

By Norman Levinson

1. A theorem of Zygmund [6] states that if

$$f(x) = \sum_1^\infty a_n e^{-\lambda_n x} \tag{1.1}$$

for $x > 0$ where $\lambda_1 > 0$ and

$$\lambda_{n+1}/\lambda_n \geq q^2 > 1, \qquad n \geq 1, \tag{1.2}$$

then

$$\sum_1^\infty |a_n| \leq A_q \int_0^\infty |f'(x)|\, dx \tag{1.3}$$

where A_q depends only on q. Here f' is the derivative of f. This theorem was extended by Waterman [4] to higher powers of $|a_n|$. This theorem is related to the high indices theorem of Hardy and Littlewood.

The high indices theorem was generalized by Levinson [2, Theorem LI] and [3, (2.06)]. In the generalized theorem e^{-x} is replaced by a function $N(x)$. The theorem may be formulated as follows:

Theorem A. *Let*

$$\sum_1^\infty a_n N(x\lambda_n) = f(x) \tag{1.4}$$

converge uniformly for $x \geq X$ for any $X > 0$. Let λ_n satisfy (1.2). *Let $N'(x) \in L(0, \infty)$ and let*

$$N(x) = -\int_x^\infty N'(t)\, dt.$$

Let

$$k(u) = \int_0^\infty N'(x) x^{-iu}\, dx. \tag{1.5}$$

Let $k(u + iv)$ be an analytic function of $w = u + iv$ in the half-plane $v > 0$ and continuous for $v \geq 0$ and let $k'(u)$ exist. Let $c = \pi/\log q$ and let A be a positive constant. Suppose that ξ is a real variable and that

$$\max_{|\xi| \leq c} \left| \frac{k(u + \xi)}{k(u)} \right| \leq e^{\theta(u)} \tag{1.6}$$

Received February 15, 1963. The preparation of this paper was supported in part by the Office of Naval Research and in part by the National Science Foundation Grant No. GP-149.