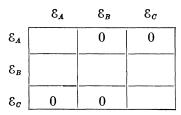
SUBINVARIANT MEASURES FOR MARKOFF OPERATORS

BY JACOB FELDMAN

1. Summary. Let $(X, \mathfrak{X}, \lambda)$ be a σ -finite measure space, and P a nonnegative contraction on $\mathfrak{L}_{\infty}(\lambda)$, such that $f_n \downarrow 0$ implies $Pf_n \downarrow 0$, λ -almost everywhere. Such operators are discussed by E. Hopf in [4], and we shall call them Markoff operators on $\mathfrak{L}_{\infty}(\lambda)$. We shall investigate the existence of P-invariant and P-subinvariant measures for such a P.

DEFINITION. A measure $\mu \prec \lambda$ is called (*sub*)-invariant or *P*-(*sub*)-invariant if $\int Pf d\mu$ (\leq) $\int f d\mu$ for all $f \in \mathcal{L}^+_{\infty}(\lambda)$. The qualifying term "on S" will be used if such an equality (or inequality) holds only for f with support in the set S, where S is a fixed set of \mathfrak{X} .

Hopf has shown how to split X into a "conservative" part C and a "dissipative" part D. In §2 we shall split D further into A and B. A will be (roughly speaking) the points of D from which one *cannot* get to C, and B = D - A. If $\mathcal{L}_{\infty}(\lambda)$ is split into $\mathcal{E}_A + \mathcal{E}_B + \mathcal{E}_C$, where $\mathcal{E}_i = \{f \mid f \text{ has its support in } i\}$, then P has reducibility properties which can be summarized by writing it as a matrix:



In §3 it is shown (Corollary a) that a *P*-subinvariant measure must be *P*-invariant on *C* (this is a generalization of a theorem of *E*. Hopf in [4] and *E*, Nelson in [5]). A corollary of this is that any *P*-subinvariant measure on \mathfrak{X} assigns measure 0 to *B*. In view of the reducibility of *P*, this shows that all we need consider are the two extreme cases of the purely conservative operator (where C = X) and the purely dissipative operator (where D = X).

For a dissipative operator P on $\mathcal{L}_{\infty}(\lambda)$, it is easy to see that there exists at least one subinvariant measure equivalent to λ (§4), although perhaps there are no invariant ones (see the example in [5]).

In 5, the "process-on-R" is studied. This device, essentially due to Halmos, was used by Harris in [3] to go from an invariant measure on a subset to a measure on the whole space. Some properties of this correspondence are needed for the subsequent sections.

For a *conservative* operator, there may well be *no* subinvariant measure. Received March 15, 1961.