A NOTE ON TORUS KNOTS AND LINKS DETERMINED BY THEIR GROUPS

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1. Introduction. The fundamental group of the complement of a link in 3-space is a powerful invariant of the link type. It is well known, however, that alone it is not sufficient to determine the link type, or even the topological type of the complementary space [5], [13], [9]. The proof of the Dehn Lemma and the sphere theorem by Papakyriakopoulos [10] immediately imply that the class of "trivial" links are determined by their groups. In fact, in the case of a knot it was for this purpose that the Dehn Lemma was originally introduced [4]. The generalization of the Dehn Lemma by Shapiro and Whitehead [11] is used in the second part of this note to show that the class of links determined by their groups may be enlarged. It will first be shown, by not unrelated considerations, that the proof by Aumann of the asphericity of alternating knots [1] implies that certain alternating knots are determined by their groups, and that there exist non-alternating knots. (A non-trivial fact already observed by Crowell in [3] and Kinoshita [6].)

2. Torus knots. Let k be an arbitrary, but fixed, non-trivial alternating knot in S^3. (An alternating knot is one which has a projection in which the crossings alternate, over and under, as one traces a path on the knot.) We assume some alternating projection has been given. Let S be a non-orientable "checkerboard" surface spanned by k. Such a surface exists, because at most one of the two checkerboard surfaces can be orientable.

Now a regular neighborhood N of the interior of S contains k on its boundary, and Aumann has proved in [1] that the maps

$$f : \pi_1(\text{boundary } N - k) \rightarrow \pi_1(S^3 - N)$$
$$g : \pi_1(\text{boundary } N - k) \rightarrow \pi_1(\overline{N})$$

are monomorphisms. (Boundary $N - k$ is connected since S is non-orientable.) It follows then by application of the van Kampen Theorem [12], that

$$\pi_1(S^3 - k) \approx \pi_1(N) *_{\pi_1(S)} \pi_1(S^3 - N).$$

Notice that $\pi_1(S)$ is free of rank, say h, so that $\pi_1(S^3 - N)$ is free of rank h. The rank is h because S contains a planar graph as a strong deformation retract, and this graph separates the plane into $h + 1$ regions. Suppose now that $\pi_1(S^3 - k)$ has a non-trivial center, then by [7; 32] $\pi_1(S)$ must have a center,

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