THE PERTURBATION OF GROUP REPRESENTATIONS

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1. Introduction. In this paper, we give an explicit method for the construction of unitary equivalences for a class of unitary representations of Abelian groups having scalar spectral measures absolutely continuous with respect to the Haar measure over the dual group. Questions of unitary equivalence are reduced to the study of a function K from the dual group to the bounded operators acting in an auxiliary Hilbert space D; for a particular character λ the range of K_λ may be regarded as the space of generalized eigenfunctions associated with λ [6]. The vector-valued Fourier transform is then used to translate perturbation hypotheses into statements about the functions K. Friedrichs (1949) [4; 402–405] indicated the utility of this method in the construction of unitary equivalences.

The main result of this paper is contained in Theorem 3, wherein conditions are given for unitary equivalence of perturbed and unperturbed representations. This result is applied to the case of an analytic perturbation in Theorems 5 and 6.

2. Preliminaries. Let H denote a separable Hilbert space, $L(H)$, the space of bounded linear operators of H into H. We consider homomorphisms R of a locally compact Abelian group G into $L(H)$ such that for each x in H, the function $g \rightarrow R(g)x$ from G to H is continuous and such that there exists a constant M for which $| R(g) | \leq M$ for all g in G. Such a homomorphism R we call a *representation* of G. Dependence upon a particular representation, where ambiguous, will be indicated by suitable indices.

If E is a Hilbert space, E' will denote the antidual of E, that is, the Hilbert space of continuous conjugate linear forms over E. The bracket $\langle \quad \rangle$ will denote the natural pairing between E' and E so that if x' and x are elements of E' and E respectively, $\langle x', x \rangle$ will denote the value of x' at x. If x and y are elements in H, then $\langle x, y \rangle$ will denote the value of x, imbedded in H' through the Riesz representation, at y; thus, $\langle x, y \rangle$ denotes the inner product of x and y in H.

We assume that an auxiliary Hilbert space D is given which, as a vector space, is dense in H and the natural map of D into H is nuclear. For the definitions and properties of nuclear maps and topological tensor products, we refer the reader to Grothendieck's thesis [5]; our notation is essentially Grothendieck's. An element a in D may be regarded as a linear functional over D through the adjoint of the natural map $D \rightarrow H$, so that if b is an element of D, we write $\langle a, b \rangle$, the inner product of a and b in H, for the value of a at b.

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