MULTIPLIER TRANSFORMATIONS, II

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1. Introduction. The present paper deals with several different topics relating to multipliers and weighted quadratic norms with negative definite weight functions.

In this section we assemble certain preliminary information needed later. Let $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_n)$ be points in n-dimensional Euclidean space E_n. If a is a real number we define $ax = (ax_1, \cdots, ax_n)$. We also define $x + y = (x_1 + y_1, \cdots, x_n + y_n)$, $x \cdot y = x_1y_1 + \cdots + x_ny_n$, and $|x| = (x \cdot x)^{\frac{1}{2}}$. If $f(x)$ is a Lesbegue measurable function on E_n, we set

$$\int_{E_n} f(x)\, dx = \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} f(x_1, \cdots, x_n)\, dx_1, \cdots, dx_n .$$

We denote by $L^2(E_n)$ the space of those Lebesgue measurable functions $f(x)$ for which

$$\left[\int_{E_n} |f(x)|^2\, dx\right]^{\frac{1}{2}} = ||f||_2$$

is finite. If $f(x) \in L^2(E_n)$ then its Fourier transform $f^\wedge(y)$ is defined as a limit in the mean of order 2,

$$f^\wedge(y) = \int_{E_n} f(x)e^{2\pi i x\cdot y}\, dx \qquad (M_2).$$

By Parseval's equality $f^\wedge(y) \in L^2(E_n)$ and $||f||_2 = ||f^\wedge||_2$. Finally the inversion formula

$$f(x) = \int_{E_n} f^\wedge(y)e^{-2\pi i x\cdot y}\, dy \qquad (M_2)$$

is valid.

Let $w(x)$ be a positive measurable function. We set

$$\mathfrak{N}_w[f]^2 = \int_{E_n} |f(x)|^2\, w(x)\, dx.$$

We further denote by $\mathfrak{N}_w(E_n)$ or $\mathfrak{N}_w$ the space of all functions $f(x)$ for which $\mathfrak{N}_w[f]$ is finite.

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