CERTAIN UPPER SEMI-CONTINUOUS DECOMPOSITIONS OF A SEMIGROUP

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We are concerned here with the problem of finding, in certain compact connected (Hausdorff) semigroups, subcontinua with prescribed properties. In particular, we seek subcontinua which are arcs and sub-semigroups.

Koch, [6], has shown that if S is a compact connected semigroup with identity, then S must contain an arc. It is shown in [5] that if $S^2 = S$, and S has a zero, then S is arcwise connected if it is either a one-dimensional continuum or is an irreducible continuum. In general, however, there need not be an arc at the identity, [5] or [6].

Our notation, in general, follows [4]. In particular, a non-empty subset L is said to be a left ideal if $SL \subseteq L$. The set of all elements x such that $x + Sx = p + Sp$ is denoted by L_p. The minimal (two-sided) ideal of S is denoted by K. If S is compact, K exists and is a retract of S. The set of idempotent elements is denoted by E. ($x \varepsilon E$ if $x^2 = x$.)

It is well known that the sets L_x form an upper semi-continuous decomposition of S when S is compact. We denote the associated hyperspace by S' and the canonical mapping by φ. I.e., $\varphi : S \to S'$, $\varphi(x) = \{L_x\}$.

If we assume that S is normal, that is to say, $xS = Sx$ for each point x, then S' is a (continuous) semigroup, under the natural multiplication and φ is a (continuous) homomorphism.

Since our methods in this note will involve S', we assume throughout that S is normal and compact.

The symbols 0 and 1 will denote the zero and identity elements respectively.

Following [12] we shall mean by an I-semigroup from a to b, an arc from a to b which is a sub-semigroup with a as zero and b as identity.

We use the term arc in the sense of [10].

If J is a two-sided ideal, we recall that the Rees quotient S mod J is the upper semi-continuous decomposition which identifies the points of J. With the natural multiplication, S mod J is a continuous semigroup with zero. The canonical mapping is, of course, one to one from $S - J$. One often makes the natural identification between $S - J$ and S mod $J - J$ mod J.

I_1 will denote the usual unit interval.

I_2 will denote the interval $[\frac{1}{2}, 1]$ with the multiplication $x \cdot y = \max(\frac{1}{2}, xy)$ where xy is the usual product. This example is due to Gleason.

I_3 will denote the unit interval with the multiplication $x \cdot y = \min(x, y)$.

Lemma 1. *Suppose that P is a subset of S'. If P is an I-semigroup each*

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