SOME SPECIAL FUNCTIONS OVER $GF(q, x)$

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1. Introduction. The function

$$\psi(t) = \sum_{r=0}^{\infty} (-1)^r \frac{t^{q^r}}{F_r}, \tag{1.1}$$

where

$$F_r = \prod_{s=0}^{r-1} (x^{q^r} - x^{q^s}), \qquad F_0 = 1,$$

is of interest in connection with the arithmetic of polynomials in $GF(q, x)$; see for example [1], [2], [4], [5], [6]. Moreover, it furnishes an interesting explicit example of an entire function in a field with a non-Archimedian valuation [5]. In particular, it possesses the factorization

$$\psi(t) = t \prod_{E} \left(1 - \frac{t}{E\xi}\right), \tag{1.2}$$

the product extending over all the non-zero elements of $GF[q, x]$. For the definition of ξ and additional properties of $\psi(t)$ see §3 below.

The object of the present paper is to define some additional functions suggested by various classical functions. We begin with an analog of the Bessel function, namely

$$J_n(t) = \sum_{r=0}^{\infty} (-1)^r \frac{t^{q^{n+r}}}{F_{n+r} F_r^{q^n}}.$$

From $J_n(t)$ we are led rather naturally to the consideration of certain other functions and classes of polynomials. For example, as a generating function for $J_n(t)$ we may mention

$$\sum_{-\infty}^{\infty} u^{q^n} J_n(t) = \psi(tG(u)),$$

where

$$J_{-n}(t) = (-1)^n \{J_n(t)\}^{q^{-n}}$$

and

$$G(t) = \sum_{r=0}^{\infty} \frac{u^{q^{-r}}}{F_r^{q^{-r}}}.$$

The definition of $G(t)$ as contrasted with that of $\psi(t)$ is rather striking; as we shall see below, it can also be thought of as an "entire" function.

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