A CONNECTED POINT SET IN THE PLANE WHICH SPIRALS DOWN ON EACH OF ITS POINTS

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The space considered here is the Euclidean plane. Let ABC be a triangle with sides AB and AC congruent, and let G be the collection of all straight line intervals having one end point at A and the other end point on the straight line interval BC. The statement that G' is an extension of G means that G' is a collection of arcs such that (1) each arc of G is a proper subset of some arc of G', (2) each arc of G' has one end point at A and contains some arc of G, and (3) no two arcs of G' have any point in common except A. The existence of an extension G' of G such that each arc of G' spirals down on one of its end points and does not spiral down on any other point has been established [4] by R. L. Moore, but G' was such that the set of all whirl points of the arcs of G' was totally disconnected. It will be shown in this paper that there exists an extension G' of G such that each arc of G' spirals down on one of its end points but on no other point and such that furthermore the set of all whirl points of the arcs of G' is a compact, connected inner limiting set which is connected im kleinen at all but a countable number of its points. In order to obtain this result some theorems on collections of spirals are used, especially theorems concerning the effect of certain homeomorphisms of the plane onto itself on equispirallic collections of spirals.

Definitions. Suppose that A and B are two points, α is an arc from A to B, J is a circle with A in its exterior and B in its interior, and n is a positive integer. The statement that α takes n steps toward spiralling down on J means that if T is a homeomorphism of the plane onto itself that leaves each point in the exterior of J fixed, then $T(\alpha)$ takes n steps toward spiralling down on $T(B)$. Suppose that A is a point and G is a collection of arcs each with one end point at A. The statement that G is equispirallic at the point P means that for each positive integer n there exists a positive number ϵ such that if J is a circle with center at P and radius less than ϵ, B is a point in the interior of J, and α is an arc of G from A to B; then α takes n steps toward spiralling down on J.

In Theorems 1, 2, and 3, H is a collection of mutually exclusive circles with mutually exclusive interiors, T is a homeomorphism of the plane onto itself which leaves each point that is in the exterior of each circle of H fixed, and A and B are points in the exterior of each circle of H.

Theorem 1. *If n is a non-negative integer and β is an arc from A to B that takes n steps and no more toward spiralling down on B, then $T(\beta)$ takes not more than $n + 2$ steps toward spiralling down on B.*

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