COVERING SPACES AND SIMPLE CONNECTEDNESS

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Introduction. The classical theory of covering spaces treated only connected and locally connected spaces (e.g., [1], [2]). Novosad recently introduced new definitions of covering spaces and also simple connectedness in terms of the covering spaces, but treated only arc-wise connected spaces. As has been pointed out by R. H. Fox in a review of Novosad's paper [5], we could define a covering space without assuming any type of connectedness for the spaces concerned (§1), in such a way that the covering spaces thus defined would be the covering spaces in the classical sense if we would further assume the connectedness and local connectedness of the spaces concerned. This idea was actually adopted in [4].

The purpose of the present paper is to give a definition of simple connectedness of a space, based on the above-mentioned idea of covering spaces, which will coincide with the classical definition of simple connectedness if the space is further assumed to be locally connected. Therefore we shall not give any new terminology to this type of simple connectedness. In order to show that our simple connectedness has all the desirable properties which the classical one had, we shall prove the lifting map theorem (§2), the monodromy theorem (§3), and the existence theorem on universal covering spaces (§5). In the first and last of these theorems, we shall not merely replace the old type of simple connectedness by the new one, but also improve the theorems themselves. We shall also define the fundamental group (or Poincaré group) of a space, having first proved the universal covering theorem (§4). In the last section we shall give an example of a simply connected space which is not locally connected, hence not simply connected in the classical sense, and we shall attempt a further generalization of simple connectedness.

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1. Throughout the paper, by a space we shall mean a Hausdorff space.

Definitions. Suppose f is a map of a space X^* into a space X. An open set U in X is called *evenly covered* by the pair (X^*, f) if and only if $f^{-1}(U)$ is the union of disjoint open sets U^* such that each U^* is homeomorphically mapped onto U under f. Under the same circumstance we shall call U^* an *even portion of* $f^{-1}(U)$ and the collection $\{U^*\}$ of U^* in $f^{-1}(U)$ a *system of even portions* of $f^{-1}(U)$. The pair (X^*, f) is called a *covering space* of X if and only if each point of X has a neighborhood which is evenly covered by (X^*, f).

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