CLASSES OF SOLUTIONS OF LINEAR SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS OF PARABOLIC TYPE

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Introduction. Consider the following system of differential equations

$$(0) \qquad \frac{\partial u_i(x, t)}{\partial t} = \sum_{j=1}^{N} \sum_{0 \le \Sigma k_s \le m} a_{ij}^{k_1 \cdots k_n}(x, t) \frac{\partial^{k_1 + \cdots + k_n}}{\partial x_1^{k_1} \cdots \partial x_n^{k_n}} u_j(x, t) + g_i(x, t)$$

$$(i = 1, \cdots, N; \qquad x = (x_1, \cdots, x_n); \qquad m = 2b > 0),$$

which may also be written in a vector form:

$$(0') \qquad \frac{\partial U}{\partial t} = P\left(x, t, \frac{1}{2\pi i} \frac{\partial}{\partial x}\right) U + G(x, t).$$

Let

$$P\left(x, t, \frac{1}{2\pi i} \frac{\partial}{\partial x}\right) = P_0\left(x, t, \frac{1}{2\pi i} \frac{\partial}{\partial x}\right) + P_1\left(x, t, \frac{1}{2\pi i} \frac{\partial}{\partial x}\right),$$

where P_0 involves only m-th order derivatives and P_1 is of order smaller than m. The system (0′) (or (0)) is *parabolic* (in the sense of Petrowski; see, for instance, [3; 59]) at the point (x, t) if, for every real vector ξ, $\xi \neq 0$, all the characteristic roots of $P_0(x, t, \xi)$ have negative real parts. If (0′) is parabolic in some domain D, then the following results hold:

A) If G and the coefficients of P are infinitely differentiable in D with respect to (x, t) then, the same holds for every solution U of (0′).
B) If G and the coefficients of P are infinitely differentiable in D with respect to (x, t) and analytic with respect to x, then the same holds for every solution U of (0′).

Special cases of A) were proved by many authors. Results which contain A) as a special case, were recently proved by Mizohata [7] and Èĭdelman [3; 74]. Theorem B) for the special case that G and the coefficients of P are independent of x was proved by Petrowski [8]; the proof of the general case was given by Èĭdelman [2], [3; 86–89].

The purpose of this paper is to show that in case A), if the coefficients of the system (0′), as functions of x, are "not far" from being analytic, then the same holds for $U(x, t)$. More precisely, we define classes of indefinitely differentiable functions, determined by the growth of their derivatives, and show, under certain assumptions on the system (0′), that if the coefficients belong to such a class, then all the solutions also belong to it. In particular we obtain a new proof for a special case of B). For the sake of simplicity, we shall first consider

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