CURVES IN MINKOWSKI SPACE

By C. C. MacDuffee

1. **Introduction.** Spaces defined by metrics which are not positive definite cannot safely be studied by methods which depend upon geometric intuition. Nor is vector analysis, the child of quaternions, very effective when the dimension exceeds three.

The present paper is an attempt to exploit the method of invariants. The group leaving the metric invariant is fundamental in this treatment. Functions invariant under this group are intrinsic or "physically real", and only such functions are herein employed except incidentally. Of course tensor analysis deals with invariants and with sets of functions whose identical vanishing is invariantive, but our approach differs from this.

Doubtless most of the material in this paper can be generalized to spaces defined by nonsingular metrics of arbitrary order and signature, but the space of special relativity is sufficiently complicated to exhibit the method, and is interesting in its own right.

2. **Minkowski space.** We shall so denote a flat space defined by the metric

$$ds^2 = \sum g_{ij}\, dx_i\, dx_j \tag{1}$$

where the g_{ij} are real numbers, and the symmetric matrix (g_{rs}) is of rank 4 and signature -2. Coördinate axes can be chosen so that

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2, \tag{2}$$

that is, so that the matrix of the quadratic form is

$$J = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

The coördinate axes are not uniquely defined by (2). Every linear transformation which leaves (2) invariant carries one such set of axes into another. The set of all linear transformations leaving (2) invariant constitutes the *total Lorentzian group*. It consists of the group of translations

$$\bar{t} = t + c_1, \qquad \bar{x} = x + c_2, \qquad \bar{y} = y + c_3, \qquad \bar{z} = z + c_4 \tag{3}$$

Received December 17, 1956.