BEHAVIOR OF CERTAIN FORCED NONLINEAR SYSTEMS OF SECOND ORDER UNDER LARGE FORCING

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1. **Introduction.** In investigating forced nonlinear systems with limiting (see §5), the author [1] obtained the following result. Let the forcing term be $Ae(t)$ where $e(t)$ has period L. Then as A becomes large, there tends to be but one periodic solution of period L. Moreover, if n is any positive integer, there is a value of A (depending on n) such that if A exceeds this value, the only solution of period nL is the aforementioned solution of period L. This shows that, given n, no proper subharmonic of period nL will exist for large enough A. It is not asserted that there is any value of A for which no subharmonic solutions exist.

In the present paper the same result is obtained for a more general case, which includes the result for systems with limiting.

2. **Periodic solutions for large-amplitude forcing.**

Theorem 1. *In the differential equation*

$$x'' + [c + f(x)]x' + kx + g(x) = Ae(t) \quad (' = d/dt) \tag{2.1}$$

let the following conditions be satisfied.

(a) *$e(t)$ is piecewise continuous and has period L.*

(b) *The constants c and k are such that the linear homogeneous equation*

$$x'' + cx' + kx = 0 \tag{2.2}$$

has no solution of period L.

(c) *$f(x)$ and $F(x) = \int_0^x f(u)\,du$ are bounded for all x, and $f(x) \to 0$ as $|x| \to \infty$.*

(d) *$g(x)$ is bounded for all x, and satisfies a Lipschitz condition for all x. Moreover, as x becomes infinite, the Lipschitz constant approaches zero,* i.e.

$$\lim_{a \to \infty} \sup_{x_2 > x_1 > a} \left| \frac{g(x_2) - g(x_1)}{x_2 - x_1} \right| = 0;$$

$$\lim_{a \to -\infty} \sup_{x_2 < x_1 < a} \left| \frac{g(x_2) - g(x_1)}{x_2 - x_1} \right| = 0.$$

(e) *If $\phi_0(t)$ is the (unique) solution of period L of*

$$x'' + cx' + kx = e(t) \tag{2.3}$$

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