THE SUMMABILITY FACTORS OF A FOURIER SERIES

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1.1 **Definitions.** Let $\sum a_n$ be a given infinite series. The series $\sum a_n$ is said to be absolutely summable (A), or summable $|A|$, if $F(x) = \sum a_n x^n$ $(|x| < 1)$ is of bounded variation in (0, 1) [18], [20]. The sum is then $\lim_{x \to 1-\lambda} F(x)$.

Let the n-th Cesàro mean of order α of the sequences $\{s_n\}$ $(s_n = \sum_{\nu=1}^n a_\nu)$ and $\{na_n\}$ be denoted by s_n^α and t_n^α respectively. The series $\sum a_n$ is said to be absolutely summable (C, α), or summable $|C, \alpha|$, $\alpha > -1$, if the series

$$\sum | s_n^\alpha - s_{n-1}^\alpha |$$

is convergent [8], [13]. By virtue of the identity

$$n(s_n^\alpha - s_{n-1}^\alpha) = t_n^\alpha \qquad [13], [14]$$

it follows that the summability $|C, \alpha|$ of $\sum a_n$ is the same thing as the convergence of the series $\sum n^{-1} | t_n^\alpha |$. It is known that summability $|C, r|$, $r > 0$, implies also summability $|A|$, but not conversely [9].

The series $\sum a_n$ is said to be strongly summable (C, k), or summable $[C, k]$, $k > 0$, to the sum s, if

$$\sum_{\nu=1}^{n} | s_\nu^{k-1} - s | = o(n),$$

as $n \to \infty$ [21].

1.2. Let $f(t)$ be a periodic function with period 2π, and integrable (L) over $(-\pi, \pi)$. Without any loss of generality we may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

$$(1.2.1) \qquad f(t) \sim \sum_1^\infty (a_n \cos nt + b_n \sin nt) = \sum_1^\infty c_n(t),$$

and

$$(1.2.2) \qquad \int_{-\pi}^{\pi} f(t)\, dt = 0.$$

Let

$$\varphi(t) = \tfrac{1}{2}\{f(x + t) + f(x - t)\}$$

and

$$\varphi^*(t) = \tfrac{1}{2}\{f(x + t) + f(x - t) - 2f(x)\}.$$

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