A CLASS OF STATIONARY PROCESSES AND A CENTRAL LIMIT THEOREM

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1. **Summary.** A certain class of stationary processes is discussed. It is shown that each process in the class has an absolutely continuous spectrum. Under some moment conditions, it is shown that such processes satisfy the central limit theorem.

2. **Introduction.** Let $\eta = (\cdots, \eta_{-1}, \eta_0, \eta_1, \cdots)$ be a doubly infinite sequence of independent identically distributed random variables. Let T be the translation operator defined on η by

$$T\eta = \eta' = (\cdots, \eta_0, \eta_1, \eta_2, \cdots). \tag{2.1}$$

Let B be the Borel field generated by η and let $g = g(\eta)$ be a function defined on B such that $E\{| g |^2\} < \infty$.

Given any such function g, we define a stationary process $\{X_n\}$ where

$$X_n = g(T^n\eta), \qquad n = 0, \qquad \pm 1, \cdots. \tag{2.2}$$

We assume without loss of generality that $E\{X_n\} \equiv 0$. Let $r_s = E\{X_n X_{n+s}\}$. Then $r_s = \int_{-\pi}^{\pi} e^{is\lambda} dF(\lambda)$, where $F(\lambda)$ is the spectral distribution function of the process. In §3 the spectral distribution function of any process of the form (2.2) is shown to be absolutely continuous. Finally it is shown in §4 that under some additional assumptions on the moment structure of the process the central limit theorem is applicable.

3. **Absolute continuity of the spectral distribution function.** In this section we prove

THEOREM I. *If $F(\lambda)$ is the spectral distribution function of a process of the form* (2.2), *$F(\lambda)$ is absolutely continuous.*

Proof. In proving the theorem we assume that each η_i is uniformly distributed on the unit interval. It is easily seen that this may be done without loss of generality. For if ξ is uniformly distributed on the unit interval, we can always construct a monotone function $\varphi(\xi)$ such that the distribution of $\varphi(\xi)$ is the same as that of any η_i, and consequently as far as the probability structure of the process $\{X_n\}$ is concerned, we may assume that each η_i is uniformly distributed [0, 1].

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