ASYMPTOTIC DEVELOPMENT OF THE SOLUTION OF DIRICHLET'S PROBLEM AT ANALYTIC CORNERS

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1. **Introduction.** In the study of certain problems of numerical analysis (cf. [3], [8]) one is led naturally to ask how the solutions of boundary value problems for partial differential equations behave as one approaches a boundary point where the boundary curves or the boundary values have a corner, i.e. a jump in the first derivative. In particular, one would like to know the order of magnitude of the various partial derivatives of the solution. The purpose of this article is to show that in the case of the ordinary Dirichlet problem for Laplace's equation in two dimensions, and for piecewise analytic boundary data, this question can be answered by an adaptation of techniques introduced by H. Lewy [7] and developed further by R. S. Lehman [4], [5]. These methods lead to infinite series of asymptotic character for the quantities in question.

Let L_1 and L_2 be two analytic arcs in the z-plane, i.e. two arcs that can be represented parametrically by two analytic functions $x + iy = z = \chi_j(s)$, $j = 1, 2$, which are, respectively, holomorphic in the closed intervals of the real s-axis that correspond to the two arcs. If $\chi_j'(s) \neq 0$ in those intervals, which will be assumed to be the case, we may stipulate, without loss of generality, that $| s |$ is the arc length. We assume further that L_1 and L_2 have one endpoint P in common, but no other point. We may choose the parameter s in such a way that L_1 corresponds to an interval $s_1 \leq s \leq 0$ and L_2 to an interval $0 \leq s \leq s_2$. It will also be assumed that P is at the origin $z = 0$.

Let $u(x, y)$ be a harmonic function in a simply connected region S bounded by L_1, L_2 and by an arc L_3 joining the points $\chi_1(s_1)$ and $\chi_2(s_2)$. Let it be further assumed that u is continuous on $L_1 + L_2$ and that its values on L_1 and L_2 are, respectively, holomorphic functions $\varphi_1(s)$, $\varphi_2(s)$ of s. We intend to study the asymptotic behavior of u and of its derivatives near the corner P under the assumption that this point is not a convex cusp, i.e. that the two curves form at P an interior angle $\alpha\pi$ with $0 < \alpha \leq 2$. Let the arcs be labeled so that the tangent to L_2 at P goes into the tangent to L_1 at P by a positive rotation through the angle $\alpha\pi$.

2. **Properties of certain asymptotic series.** We shall make extensive use of elementary properties of certain generalized asymptotic series representations that have been introduced in [7], [4] and [5]. The proofs of most of these proper-

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