# THE DIAGONALIZATION OF SKEW-HERMITIAN MATRICES

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**1. Introduction.** Hermitian matrices may be diagonalized by an infinite sequence of unitary transformations [2]. Each of these transformations may be chosen in such a manner as to annihilate a pair of selected off-diagonal elements. In the diagonalized form, the elements represent the characteristic values of the Hermitian matrix.

This procedure, originally due to Jacobi, has lately been used widely for the practical simultaneous determination of the characteristic values; see, for example, Givens [1].

The purpose of this paper is to illustrate that all skew-Hermitian matrices may be diagonalized in the same manner, with a rate of convergence that is the same as that for the Hermitian case. It follows that this technique represents also a practical way of accomplishing the diagonalization.

**2. The sequence of transforms.** It is desired to determine the characteristic values of the $n$-dimensional matrix

$$A = \begin{bmatrix} a_{11} & & & & a_{1n} \\ & \ddots & & & \\ & & a \cdots b & & \\ & & \vdots \ddots \vdots & & \\ & & c \cdots d & & \\ & & & \ddots & \\ a_{n1} & & & & a_{nn} \end{bmatrix}.$$

of arbitrary complex elements $a_{ij}$ . Let $a$, $d$ be the elements of $A$ in the main diagonal locations $(i, i)$ and $(j, j)$, $i \neq j$, respectively, and $b$, $c$ the elements in the locations $(i, j)$ and $(j, i)$.

A suitable unitary transformation then is

$$T = \begin{bmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & & t \cdots -(1 - r^2)^{\frac{1}{2}} & & \\ & & \vdots \ddots \vdots & & \\ & & (1 - r^2)^{\frac{1}{2}} \cdots \bar{t} & & \\ & & & \ddots & \\ 0 & & & & 1 \end{bmatrix},$$

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