SOLUTION OF THE LINEARIZED BOLTZMANN TRANSPORT EQUATION FOR THE SLAB GEOMETRY

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1. In a previous paper [4], we determined the spectrum of the operator

$$A\cdot = -\mu \frac{\partial \cdot}{\partial x} + \frac{c}{2} \int_{-1}^{1} \cdot d\mu', \tag{1.1}$$

defined over a certain Hilbert space. This operator, which is not self-adjoint, arises in connection with a linearized form of the Maxwell-Boltzmann equation appropriate to the transport of neutrons in an infinite slab surrounded by a perfect absorber, when certain simplifying assumptions are made. The initial value problem for the equation in question may be written as

$$\begin{aligned} &u_t = Au, \qquad u = u(x, \mu, t), \qquad |x| \le a, \qquad |\mu| \le 1, \qquad t > 0 \\ &u(\pm a, \mu, t) = 0, \qquad \mu \lessgtr 0, \qquad t > 0; \qquad u(x, \mu, 0) = f(x, \mu) \end{aligned} \tag{1.2}$$

where

$$t = vt', \qquad u(x, \mu, t) = e^{\sigma v t'} N(x, \mu, t').$$

Here t' is the time, v is the neutron speed, x is the position coordinate perpendicular to the sides of the slab, σ is the total cross section, $c/\sigma > 0$ is the average number of neutrons emerging from the collision of a neutron with a nucleus, and N is the density of the neutron beam in directions with x-direction cosine μ. It is assumed that v, σ, and c are constant. The boundary conditions come from the fact that the slab is immersed in a perfect absorber.

Originally it was thought that the operator A possessed a complete set of eigenfunctions, in terms of which the solution to the initial value problem could be expanded. In [4] we showed this was not the case; the set of eigenfunctions is finite (but not empty).

In the present paper we solve the initial value problem by an application of semi-group theory. The operator A satisfies the conditions of the Hille-Yosida Theorem and so generates a semi-group of operators $T(t)$, $t \ge 0$, and $u(x, \mu, t) = T(t)f$ solves the problem (1.2). Using the Laplace integral and shifting the path of integration, we obtain

$$\begin{aligned} u &= \sum_{j=1}^{m} (f, \psi_j^*)\psi_j e^{\beta_j t} + \lim_{\omega \to \infty} \frac{1}{2\pi i} \int_{\gamma - i\omega}^{\gamma + i\omega} e^{\lambda t} R(\lambda, A) f \, d\lambda, \\ &= \sum_{j=1}^{m} (f, \psi_j^*)\psi_j e^{\beta_j t} + \zeta(x, \mu, t), \end{aligned} \tag{1.3}$$

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