MEROMORPHIC FUNCTIONS WITH ASSIGNED ASYMPTOTIC VALUES

By Maurice Heins

1. It was shown (essentially) by Mazurkiewicz [6] that the set of asymptotic values of a meromorphic function is an analytic set. Subsequently Kierst [5] showed that a given analytic set of the extended plane could be realized as the set of asymptotic values of a suitably chosen meromorphic function having the interior of the unit circle as its domain. Recently the author [4] established the existence of an entire function whose set of asymptotic values is a given analytic set of the extended plane containing the point at infinity. An immediate consequence of this result is the existence of a meromorphic function whose domain is the finite plane and whose set of asymptotic values is an assigned analytic set of the extended plane.

It is to be noted that the domains of the examples of Kierst (see Note 1) and the author constitute extreme cases of non-compact Riemann surfaces. The problem arises *whether, for a given non-compact Riemann surface, F there exists a meromorphic function whose domain is F and whose set of asymptotic values is an assigned analytic set of the extended plane.* The object of the present note is to show that this is in fact the case. Simultaneously we shall see that *there exists an analytic function with domain F whose set of asymptotic values is an assigned analytic set of the extended plane containing the point at infinity.*

From these results it then follows in an obvious way that, *if G is a Riemann surface whose universal covering surface is not hyperbolic, there exists a conformal map of F into G whose set of asymptotic points* (in G) *is an assigned analytic set of G.* The situation for the case where the universal covering surface of G is hyperbolic appears to be fairly recondite.

2. Our problem may be reduced to establishing the existence of an analytic function with domain F which does not possess any finite asymptotic values. For suppose that f is such an analytic function and let g denote a function meromorphic in the finite plane whose set of asymptotic values is a given analytic set E_1 of the extended plane (or an entire function whose set of asymptotic values is a given analytic set of the extended plane, E_2 , containing the point at infinity). Then $g \circ f$ yields a function with domain F which in the first instance is meromorphic and has E_1 as its set of asymptotic values and in the second instance is analytic and has E_2 as its set of asymptotic values.

3. Actually we shall prove the following stronger result:

There exists an analytic function f whose domain is F and which has the property that for each bounded region Ω of the finite plane each component of $f^{-1}(\Omega)$ is relatively compact.

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